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# THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
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## JUNE 1922

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THE
ASTROPHYSICAL JOURNAL
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VOLUME LV JUNE 1922 NUMBER 5

# PARTIAL EXPLANATION, BY WAVE-LENGTHS, OF THE K-TERM IN THE B-TYPES

By SEBASTIAN ALBRECHT

ABSTRACT

*K-term of the radial velocity of Class B stars.*—Whether the K-term is due to an expanding universe, to causes within the stars or to the Einstein effect, it is of importance to determine *the part due to the use of inaccurate normal wave-lengths* in the reductions. For most of the lines upon which the B-type velocities are based, such as those of Si and He, sufficiently accurate wave-lengths are not available; but in the case of twenty O and N lines, new wave-lengths by Clark are found to be systematically 0.063 A longer than the adopted normals. A discussion of the available portions of the data on which the K-term is based, indicates that the new wave-lengths for this one group of lines alone would reduce the K-term, which is about 4 km, by about 0.3 km for the entire B class and by about 0.8 km for classes Bo–B2.

*Test of the relative accuracy of groups of wave-length normals.*—Since the spectra of different classes of stars are radically different, it is *suggested* that a comparison of the mean wave-lengths for the lines which these classes have in common might give evidence as to the relative accuracy of the principal group of normals involved in each case. For instance the wave-lengths of types Bo–B2 are based more on O, N, and Si lines than those of types B3–B5 and they are systematically 0.015 A shorter.

In 1911 Campbell[1] found that, after the radial component of the sun's motion in space had been deducted from the radial velocity of each star, the remaining portions of the radial velocities were not at random but in every part of the sky showed a systematic average residual directed away from the observer. This residual, which has become known as the K-term, appeared to be essentially the same for all limited areas of the sky, but to differ according to the spectral class of the star. Interpreted as a phenomenon of

[1] *Lick Observatory Bulletins*, **6**, 107 and 127, 1911.

stellar motion, it means that the universe, represented by the stars employed by Campbell, is an expanding one, the rate of expansion depending upon the spectral class. Thus the stars of types B and M are systematically moving away in all directions from the position occupied at any instant by the solar system with a speed of about 4 km per second. The A and K types are moving outward with speeds of about 1 and 2 km per second respectively, while the F and G types show a K-term which is essentially zero.

However, the K-term could be produced, as was also pointed out by Campbell, by the use of wave-lengths which are on the average too low. Thus if it could be assumed that the wave-lengths which were employed in the reductions for radial velocity for the B types were too small by about 0.06 A, then the entire K-term for these types would disappear. Campbell was inclined toward the view that such changes in the wave-lengths may be due to causes within the stars, such as high pressures, absorption through a wide range of depth in the stellar atmospheres and, with particular reference to helium, conditions in the stars such as to augment considerably the red components of lines which occur in close pairs. Freundlich[1] suggested that, due to the Einstein effect, all stellar lines are displaced toward the longer wave-lengths relatively to terrestrial comparison spectra. Such displacements, which would merge with displacements due to radial velocity and reappear in the K-term, would differ for stars of the different spectral classes due to mean differences in their masses. Although these factors must not be disregarded, nevertheless the laboratory values themselves require close scrutiny.

At about the same time that the K-term was found, attention was called by the writer to the unsatisfactory state of our knowledge in regard to the laboratory values of the wave-lengths upon which the radial velocities for the B-type depend.[2] This is especially true for the three prominent and important silicon lines $\lambda$ 4552, $\lambda$ 4567, and $\lambda$ 4574, for which the two most accurate laboratory determinations differ by an average of 0.09 tenth meters, corresponding to a systematic difference of 6.0 km in the

[1] *Astronomische Nachrichten*, **202**, 17, 1915.

[2] *Publications of the Astronomical and Astrophysical Society of America*, **2**, 71, 1911.

radial velocities derived from them. There has been only one response, I believe, in the ten years which have elapsed, to the appeal which was made for additional laboratory determinations. Mr. Clark,[1] under the direction of Professor Fowler, has redetermined the wave-lengths for the oxygen and nitrogen lines. The effect of the new wave-lengths upon the radial velocities and the K-term can be approximated.

TABLE I

| λ Clark (Rowland System) | λ Normal (F. and A.) | λ Clark − λ Normal | Change in Radial Velocity | Element |
|---|---|---|---|---|
| | | A | km | |
| 4317.327 | .272 | +0.055 | −3.8 | O |
| 19.814 | .762 | + .052 | −3.6 | O |
| 45.738 | .677 | + .061 | −4.2 | O |
| 48.246 | .134 | + .112 | −7.7 | N |
| 49.602 | .541 | + .061 | −4.2 | O |
| 51.443 | .495 | − .052 | +3.6 | O |
| 67.074 | .012 | + .062 | −4.3 | O |
| 4415.057 | .076 | − .019 | +1.3 | O |
| 17.143 | .121 | + .022 | −1.5 | O |
| 4447.205 | .163 | + .042 | −2.8 | N, O |
| 4591.158 | .066 | + .092 | −6.0 | O |
| 96.365 | .291 | + .074 | −4.8 | O |
| 4621.582 | .548 | + .034 | −2.2 | N |
| 30.729 | .703 | + .026 | −1.7 | N |
| 39.043 | .937 | + .106 | −6.9 | O |
| 42.006 | .886 | + .120 | −7.8 | O |
| 49.327 | .250 | + .077 | −5.0 | O |
| 51.032 | .925 | + .107 | −6.9 | N, O |
| 61.829 | .728 | + .101 | −6.5 | O |
| 4676.426 | .291 | + .135 | −8.7 | O |
| Means (20 lines) | ..... | +0.063 | −4.2 | ..... |

Table I, which is self-explanatory, contains such of the oxygen and nitrogen lines as were included in the reductions for radial velocity. For the 20 lines listed the wave-lengths by Clark are on the average higher by 0.063 A, which corresponds to a change of −4.2 km in the radial velocities derived from them.

The details of the radial velocities from which Campbell derived the K-term have not been published, so that it is not possible to determine the exact part played by any particular group of lines in the reduction of his 225 B-type stars. However, this can be

[1] *Astrophysical Journal*, **40**, 332, 1914.

very roughly approximated with the aid of two partial sets of data for which such details are available, namely, the measures of 20 Orion-type stars by Frost and Adams[1] and the B-type stars contained in Volume IX of the *Publications of the Lick Observatory*. For these data the changes in the radial velocities produced by the substitution of the wave-lengths of Clark in place of the normal wave-lengths which were employed are summarized in Tables II and III. Tables IV and V show, for the smaller subdivisions of

TABLE II

CHANGES IN RADIAL VELOCITY PRODUCED BY WAVE-LENGTHS OF CLARK. MEASURES BY FROST AND ADAMS

| STAR | TYPE | CHANGE FOR O AND N LINES | | CHANGE FOR STAR | | RELATIVE REPRESENTATION OF THE O AND N LINES IN THE MEASURES (IN PER CENT) |
|---|---|---|---|---|---|---|
| | | km | Plates | km | Plates | |
| $\epsilon$ Orionis | Bo | −6.9 | 1 | −0.1 | 7 | 22 |
| $\zeta$ Orionis | Bo | −1.5 | 3 | −0.2 | 7 | 13 |
| $\kappa$ Orionis | Bo | −1.4 | 7 | −0.2 | 10 | 15 |
| $\zeta$ Persei | B1 | −2.5 | 7 | −0.7 | 7 | 31 |
| $\beta$ Can. Maj. | B1 | −2.2 | 5 | −1.0 | 5 | 42 |
| $\epsilon$ Can. Maj. | B1 | −2.2 | 4 | −0.7 | 4 | 33 |
| $\gamma$ Pegasi | B2 | −0.4 | 12 | −0.2 | 12 | 22 |
| $\zeta$ Pegasi | B2 | −0.5 | 3 | −0.1 | 6 | 8 |
| $\gamma$ Orionis | B2 | −2.9 | 6 | −0.4 | 10 | 9 |
| 102 Herculis | B2 | −0.1 | 2 | 0.0 | 5 | 6 |
| $\iota$ Herculis | B3 | +1.3 | 1 | 0.0 | 6 | 1 |
| $\eta$ Lyrae | B3 | −1.5 | 1 | 0.0 | 6 | 2 |
| $\zeta$ Draconis | B5 | −1.5 | 2 | −0.1 | 6 | 5 |
| $\epsilon$ Delphini | B5 | −0.1 | 2 | 0.0 | 4 | 8 |
| $\beta$ Orionis | B8p | −1.5 | 1 | 0.0 | 24 | 0 |

Class B, the relative representation of the different groups of lines in the measures. As far as can be judged from Tables II to VI the inference seems to follow that the new wave-lengths of Clark for oxygen and nitrogen will reduce the K-term for the entire Class B by about 0.3 km, and for Bo–B2 by about 0.7 or 0.8 km.

It does not seem unlikely that additional portions of the K-term may have their origin in the normal wave-lengths of other groups of lines. In this connection it may not be amiss to recall

[1] *Publications of the Yerkes Observatory*, **2**, 143, 1904.

TABLE III

CHANGES IN RADIAL VELOCITY PRODUCED BY WAVE-LENGTHS OF CLARK MEASURES IN *Publications of Lick Observatory*, 9

| Star | Type | Change for O and N Lines | Change for Star | Relative Representation of the O and N Lines in the Measures | Measured by |
|---|---|---|---|---|---|
| | | km | km | Percentage | |
| $\tau$ Scorpii | Bo | −2.0 | −0.9 | 44 | A |
| | | −2.0 | −1.2 | 56 | P |
| $\beta$ Crucis | B1 | −1.7 | −0.8 | 45 | A |
| | | −1.9 | −1.0 | 53 | P |
| $\alpha$ Pyxidis | B2 | −2.1 | −1.0 | 46 | A |
| $\alpha$ Lupi | B2 | −1.8 | −1.1 | 60 | A |
| | | −1.9 | −1.0 | 52 | P |
| $\alpha$ Pavonis | B3 | −1.6 | −0.3 | 15 | A |

A = S. Albrecht P = H. K. Palmer

TABLE IV

FROM MEASURES BY FROST AND ADAMS

| Type | No. of Stars | H$\gamma$ | Mg | He | Si | O and N | Metals |
|---|---|---|---|---|---|---|---|
| | | Percentage | Percentage | Percentage | Percentage | Percentage | Percentage |
| Bo | 3 | 15 | 4 | 58 | 10 | 11 | 0 |
| B1 | 3 | 3 | 3 | 23 | 33 | 36 | 1 |
| B2 | 4 | 5 | 14 | 42 | 26 | 11 | 1 |
| B3 | 2 | 2 | 30 | 54 | 12 | 1 | 0 |
| B5 | 5 | 12 | 35 | 47 | 4 | 2 | 1 |
| B8 | 2 | 8 | 29 | 28 | 1 | 0 | 35 |
| Bo–B8 | 19 | 8 | 19 | 43 | 14 | 11 | 5 |

TABLE V

FROM *Publications of the Lick Observatory*, 9

| Type | No. of Stars | H$\gamma$ | Mg | He | O and N | C | Metals |
|---|---|---|---|---|---|---|---|
| Bo | 1 | 5 | 6 | 16 | 50 | 5 | 18 |
| B1 | 1 | 15 | 5 | 32 | 33 | 10 | 5 |
| B2 | 6 | 33 | 4 | 39 | 13 | 9 | 3 |
| B3 | 9 | 30 | 14 | 42 | 1 | 12 | 1 |
| B5 | 2 | 0 | 32 | 68 | 0 | 0 | 0 |
| B8 | 2 | 64 | 14 | 14 | 0 | 6 | * |
| Bo–B9 | 21 | 29 | 12 | 39 | 8 | 9 | 2 |

* A number of unidentified lines were not included in the percentages for B8.

that systematic differences between Lick and Bonn radial velocities[1] for types F–M, of the same order of magnitude as the K-term for these types, have been found to be due to differences in the normal wave-lengths which were employed in the reductions. The normal wave-lengths for the lines in Class B are perhaps not as accurately known as those for types F–M. I wish again to urge the desirability of additional laboratory study of the other groups of lines which occur in Class B stars, and especially those of silicon and helium. In view of the importance of the problem and the

TABLE VI

DATA FROM WHICH THE K-TERM WAS DETERMINED

| Type | No. of Stars | Part of Total Number (in per cent) | |
|---|---|---|---|
| B0 | 13 | 6 | |
| B1 | 17 | 7 | 24 |
| B2 | 25 | 11 | |
| B3 | 72 | 32 | |
| B4 | 2 | 1 | 56 |
| B5 | 51 | 22 | |
| B8 | 22 | 10 | |
| B9 | 21 | 10 | 20 |
| A0* | 2 | 1 | |
| B–A | 225 | 100 | |

* Two of Campbell's stars seem to have been reclassified as A0 at Harvard.

apparent difficulties with the laboratory spectra, a certain amount of duplication would not be wasted effort.

If Tables IV and V may be regarded as approximately representative for the B types, they may also serve as a basis for a somewhat coarser grouping of the subdivisions of Class B for certain statistical purposes. B. Boss pointed out that the proper motions of stars of Classes B8–B9 more nearly resemble those of A type than those of early B type, and that therefore the B8–B9 stars should be grouped with early A types rather than with early B types. These results were confirmed by Campbell, and my Table IV is also in accord with this conclusion, the abruptly large

[1] *Astrophysical Journal*, **40**, 475, 1914.

representation of the metallic lines being the principal distinguishing characteristic. Additional subdivisions indicated are Bo–B2 and B3–B5. In the former silicon and oxygen are strongly represented in the measures and magnesium only slightly, while in the latter the reverse is true.

The low values of the normal wave-lengths for the oxygen and nitrogen lines contribute toward the progression in K which is indicated within the B types. Campbell's 225 stars are probably inadequate, under present limitations, for accurate solutions for subdivisions Bo–B2, B3–B5, and B8–B9. Campbell (*op. cit.*, p. 123) finds K = +4.7 km per second for stars of Class Bo–B5 and only +4.1 km for the entire class Bo–B9. His Class Bo–B5 comprises 80 per cent (see Table VI) and his Class B8–B9 only 20 per cent of his list. Campbell also finds K = +1.62 km for 222 stars of Classes B8–A9 and K = +0.95 km for 177 stars of Classes Ao–A9 alone. On the basis of proportionality the former results would give approximately K = +2 km, while the latter results would give K = +4 km for classes B8–B9. Perhaps a K-term of about +3 km for B8–B9 will fairly well represent the actual facts for the data concerned. In view of the strong representation of the oxygen and nitrogen lines in Bo–B2 and their practical absence in the measures of stars B3–B5, K is probably greater in the former than in the latter subdivisions.

The observed wave-lengths for the stellar lines are capable of contributing, so to speak, circumstantial evidence toward the relative correctness of the adopted normal wave-lengths for different groups of lines. A general discussion of this nature will be more worth while after the laboratory has contributed more nearly its full share toward this problem. For the present I shall be content with merely indicating how low values of the normal wave-lengths for the oxygen and nitrogen lines will impress a corresponding effect upon the observed stellar wave-lengths. The subdivisions Bo–B2 and B3–B5 are suitable for this purpose, because the oxygen and nitrogen lines are strongly represented in the former and nearly absent in the latter. If the normal wave-lengths for the oxygen and nitrogen lines, constituting nearly 20 per cent of the lines actually employed in Bo–B2, averaged 0.056 A too low, the

radial velocities for these lines will average 3.7 km too high, making the mean radial velocities for the stars about 0.74 km too high. In the residuals of velocity the oxygen and nitrogen lines will show up as 2.96 km too high, and the remaining 80 per cent of the lines as 0.74 km too low. Thus the observed wave-lengths for the oxygen and nitrogen lines will be brought up four-fifths of the way toward where they should be, while the other 80 per cent of the lines will have their observed wave-lengths lowered from where they should be by an amount equal to one-fifth of the average error in the adopted normal wave-lengths for oxygen and nitrogen. Thus all the observed wave-lengths on the plates will finally be 0.011 A too low. Therefore, considering only the effects due to the oxygen and nitrogen lines, all the observed wave-lengths in classes B0–B2 should, on the average, be 0.011 A lower than in classes B3–B5. Table VII may be of interest in this connection, even though the

TABLE VII

OBSERVED WAVE-LENGTHS IN CLASSES B0–B2 AND B3–B5

| LINE | NOR. MILLS λ 4500 CENTRAL | | FROST AND ADAMS λ 4500 CENTRAL | | SO. MILLS λ 4340 CENTRAL | | β CAN. MAJ. λ 4500 CENTRAL |
|---|---|---|---|---|---|---|---|
| | B0–B2 | B3–B5 | B0–B2 | B3–B5 | B0–B2 | B3–B5 | B1 |
| 4267.3 C | ..... | ..... | ..... | ..... | $0.260_{14}$ | $0.278_{18}$ | ..... |
| 4340.6 Hγ | $0.618_{30}$ | $.643_{16}$ | $0.622_{30}$ | $0.642_{16}$ | $.625_{15}$ | $.632_{18}$ | $.609_{29}$ |
| 4388.1 He | $.073_{43}$ | $.109_{6}$ | $.094_{37}$ | $.099_{19}$ | $.109_{15}$ | $.109_{18}$ | $.082_{33}$ |
| 4437.7 He | ..... | ..... | $.713_{26}$ | $.726_{13}$ | $.727_{12}$ | $.725_{15}$ | $.721_{22}$ |
| 4471.6 He | $.664_{46}$ | $.685_{14}$ | $.668_{39}$ | $.689_{22}$ | $.684_{15}$ | $.707_{18}$ | $.672_{34}$ |
| 4481.4 Mg | $.365_{42}$ | $.375_{13}$ | $.376_{33}$ | $.397_{22}$ | $.368_{15}$ | $.376_{18}$ | $.362_{34}$ |
| 4552.7 Si | $.752_{47}$ | $.798_{8}$ | $.769_{34}$ | $.752_{8}$ | ..... | ..... | $.753_{34}$ |

wave-lengths given are not strictly definitive and the effects of errors in the normal wave-lengths of other groups of lines are also involved. The subscripts represent provisionally adopted weights, based partly upon the number of plates and partly upon the number of stars. Of the seventeen comparisons listed, fourteen give the wave-length smaller in B0–B2 than in B3–B5 and only two give it as larger. The mean difference in wave-length is 0.015 A, not all of which is necessarily due to the oxygen and nitrogen lines.

The results in Tables II–V are probably fairly representative for the dispersion of three prisms. For several reasons, among them the broad character of the spectrum lines in many B-type stars, a considerable number of the radial velocities are based on the lower dispersions of one and two prisms. The effects of systematic differences between results obtained with dispersions of one, two, and three prisms will also have to be evaluated. The details required for this purpose have not been published. However, unless my memory is in error on this point, the radial velocities obtained with the lower dispersions showed a systematic tendency toward larger positive values than those obtained with three prisms. Any uneliminated portions of such systematic differences will directly contribute toward the K-term.

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DUDLEY OBSERVATORY
ALBANY, NEW YORK
January 1922

# EFFECTIVE WAVE-LENGTHS OF 129 STARS WITHIN 0°.5 OF THE NORTH POLE AND OF THE Σ COMPANION OF POLARIS[1]

By EJNAR HERTZSPRUNG

ABSTRACT

*Effective wave-lengths, color indices, and magnitudes of 129 stars within 0°.5 of the North Pole.*—The results of measurements of about 500 images on 12 plates (Lumière Σ) taken in 1912 with the 60-inch reflector and a grating whose constant was 6 mm, are given in Table II. In determining the effective wave-length $\lambda_{eff}$, corrections were made (1) for diameters differing from the normal diameter 0.10 mm and (2) to reduce to zenith transmission. For each star the effective color index $I_\lambda$ was obtained from $\lambda_{eff}$ by use of the formula: $I_\lambda = 0.005(\lambda_{eff} - 4216\text{ A}) = 19.721(\lambda_{eff} - 1.069\text{ mm})$. The star, *Harvard Annals*, **48**, No. 85, is probably wrongly included in the series of red stars of the North Polar Sequence. For the Σ *companion to Polaris*, $I_\lambda$ was found equal to $+0.11 \pm 0.07$, which corresponds to a spectrum of type A0 to A5.

*Relation between effective color-index and the color-index of Seares.*—For 76 stars both $I_\lambda$ and $I_{MW}$ (the difference between photovisual and photographic magnitudes) were available for comparison. It was found that $I_\lambda = (0.90 \pm 0.05) I_{MW} - (0.10 \pm 0.05)$, a very satisfactory result.

*Relation between photographic magnitude and diameter of image* was found to be: $m = \text{Const.} - 34d + 35d^2$, for $d$ between 0.05 and 0.30 mm.

During my stay at Mount Wilson in 1912 the plates of the North Polar region listed in Table I were taken with the 60-inch reflector for the determination of effective wave-lengths.[2] Lumière Σ plates were used. The focal length of the instrument is 7606 mm and the grating constant was exactly 6 mm.

The co-ordinates of the center of the field are given in the system of *Harvard Annals*, **48**, 21, Table VI, sixteenth and seventeenth columns. The efficiency of each plate is indicated by the photographic magnitude of stars whose central image has a diameter of one-tenth of a millimeter. The total number of images measured is somewhat greater than that used in the present note because some stars showing only very faint images were finally rejected. The effective wave-lengths of hazy images were given half-weight. The mean weight-factor of a plate, which is given in the last column

[1] *Contributions from the Mount Wilson Observatory*, No. 231.

[2] Compare *Mt. Wilson Contr.*, Nos. 100, 101; *Astrophysical Journal*, **42**, 92, 111, 1915.

of Table I, may therefore be taken as a measure of the mean quality of the images. The quality 0.50 thus means that all images of the plate were given half-weight.

TABLE I

PHOTOGRAPHS OF THE NORTH POLE

| Plate No. | Date 1912 | Aperture (inches) | Sidereal Time at Middle of Exposure | Exposure Time (min.) | Co-ordinates of Center of Field | | Efficiency of Plate | Number of Images Measured | Quality of Images |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $x$ | $y$ | | | |
| 4..... | July 19 | 60 | $20^h\ 5^m$ | 60 | $-480''$ | $+\ 70''$ | $14^m.8$ | 32 | 0.64 |
| 5..... | ..... | 40 | 22 23 | 95 | $-520$ | $+\ 30$ | 14.6 | 108 | .70 |
| 6..... | ..... | 40 | 23 26 | 9.5 | $-510$ | $+\ 50$ | 12.0 | 15 | .83 |
| 10..... | July 20 | 40 | 21 1 | 9.5<br>3 | $-180$<br>$-350$ | $+\ 50$ | 11.9<br>10.9 | 29 | .83 |
| 11..... | ..... | 40 | 22 36 | 95 | $-190$ | $+\ 50$ | 14.6 | 112 | .69 |
| 14..... | July 21 | 40 | 18 16 | 31.3 | $+\ 10$ | $+140$ | 13.3 | 44 | .50 |
| 15..... | ..... | 40 | 19 15 | 30 | $-\ 40$ | $+\ 90$ | 13.7 | 34 | .56 |
| 16..... | ..... | 40 | 19 53 | 10.5<br>1, 0.3 | $-60, +20$<br>$+120, +190$ | $+\ 60$ | 12.7, 11.7<br>10.7, 9.7 | 27 | .52 |
| 17..... | ..... | 60 | 21 48 | 60 | $-200$ | $+\ 30$ | 14.9 | 23 | .50 |
| 204..... | September 14 | 40 | 21 7 | 6 | $-120$ | $-\ 10$ | 11.6 | 14 | .79 |
| 205..... | ..... | 40 | 21 43 | 60 | $-\ 80$ | $-\ 50$ | 13.8 | 44 | .56 |
| 206..... | ..... | 40 | 22 29 | 19 | $-\ 60$ | $-\ 70$ | 12.7 | 29 | 0.83 |

The diameter of the field measured was $1150''$ when the full aperture was used, and $2500''$ when the mirror was diaphragmed down to 40 inches. The plates were measured and reduced at Potsdam. At the same time a series of plates of the Pleiades has been discussed, which afford 9972 effective wave-lengths. This last material has been used for a more thorough determination of the variation of the effective wave-length with the intensity of the image. The effective wave-lengths were measured in millimeters by moving the plateholder with a micrometer screw and reading to $\frac{1}{2}\ \mu$. The diameter of the central image of the star was taken as a measure of the intensity of the spectra of first order. As normal strength that shown when the diameter of the central image is 0.1 mm or $100\ \mu$ was chosen. For other diameters of the central image the following corrections to the normal diameter were found:

| Diameter of central image $d=$ | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 mm |
|---|---|---|---|---|---|---|
| Correction to $d=0.10$ mm | $+13$ | $+10$ | $+7$ | $+4$ | $+2$ | $+0\ \mu$ |

| 0.11 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 | 0.17 | 0.18 | 0.19–0.24 mm |
|---|---|---|---|---|---|---|---|---|
| $-2$ | $-3$ | $-4$ | $-5$ | $-6$ | $-6$ | $-7$ | $-7$ | $-7\mu$ |

No conspicuous difference in the corrections for stars of different effective wave-lengths was noted.

These corrections were also used in reducing the effective wave-lengths given in the present note. The correction required to reduce the effective wave-lengths to the zenith owing to selective extinction of light in the earth's atmosphere amounts to $-23$ A or $-6\,\mu$ for the North Polar region as seen from Mount Wilson.

To convert the effective wave-lengths expressed in millimeters into Angstrom units they must be multiplied by 3944.2. It will be more convenient for comparison with determinations of the difference between photographic and visual magnitudes, however, to transform the effective wave-length measured in mm directly into the equivalent values of color index.

I have found that a difference of $1^M$ in the color index in the system of $I_H$ of the Göttingen Actinometry, Part B, corresponds nearly to a difference of 200 A in the effective wave-length. By adopting this and comparing the color indices $I_H$ of stars occurring in the Göttingen Actinometry with their effective wave-lengths on other plates taken with the 60-inch reflector it was found that $I_H = 0$ corresponds to an effective wave-length of 1.069 mm or 4216 A. Hence, indicating the color index calculated from the effective wave-length by $I_\lambda$, we have

$$I_\lambda = {}_2{}^1{}_0(\lambda_{\text{eff}} - 4216\text{ A}) = 19.721(\lambda_{\text{eff}} - 1.069\text{ mm})$$

The accuracy of the effective wave-length varies with the intensity of the image. Different weights were therefore assigned to the effective wave-lengths, according to the diameter of the central image. These weights were decreased for images which were not of normal strength, because of the uncertainty of the reduction of the effective wave-length to $d = 0.10$ mm, which may vary with the quality of the image. The following weights of $I_\lambda$ were finally adopted for different values of the diameter $d$ of the central image.

| $d=$ | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 | 0.13 mm |
|---|---|---|---|---|---|---|---|---|---|
| $1/\lambda_2(I_\lambda) = 1/(\text{m.e.})^2$ | 2 | 4 | 8 | 18 | 28 | 38 | 48 | 56 | 58 |

| 0.14 | 0.15 | 0.16 | 0.17 | 0.18 | 0.19 | 0.20 | 0.21 | 0.22 | 0.23 | 0.24 mm |
|---|---|---|---|---|---|---|---|---|---|---|
| 52 | 44 | 36 | 28 | 22 | 16 | 12 | 8 | 6 | 4 | 2 |

For example, for $d=0.19$ mm, the adopted weight is 16, corresponding to a mean error in the color index $I_\lambda$ of $\pm\sqrt{1/16}=\pm 0.25$. As stated above, for bad (hazy) images these weights were divided by 2. In fact the North Polar plates here employed are perhaps the poorest I took at Mount Wilson.

The results are given in Table II. The first column contains the number of the star taken from the *Harvard Annals*, **48**, 21, Table VI, and the second, the number of the North Polar sequence. The third column gives the photographic magnitude as derived from my estimates of the diameter of the central image. These magnitudes, according to which the stars have been arranged in the table, are only approximate. No correction for distance from the center of the field has been applied. The scale of Seares[1] was taken as a standard. The following relation between magnitude and diameter $d$ of the central image was found.

| $d=$ | | | | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 mm |
|---|---|---|---|---|---|---|---|---|---|
| $m+$plate constant$=$ | | | | 16.0 | 15.9 | 15.8 | 15.5 | 15.2 | 14.8 |
| 0.10 | 0.11 | 0.12 | 0.13 | 0.14 | 0.15 | 0.16 | 0.17 | 0.18 | 0.19 mm |
| 14.6 | 14.3 | 14.0 | 13.8 | 13.6 | 13.3 | 13.1 | 12.9 | 12.7 | 12.5 |
| 0.20 | 0.21 | 0.22 | 0.23 | 0.24 | 0.25 | 0.26 | 0.27 | 0.28 | 0.29 mm |
| 12.2 | 12.0 | 11.9 | 11.6 | 11.5 | 11.4 | 11.2 | 11.0 | 10.9 | 10.7 |

The fourth column contains the color index $I_\lambda$ as derived from the effective wave-length, and the fifth column, its weight, i.e., the reciprocal square of the mean error. The sum of the weights of $I_\lambda$ for all 129 stars is 10,281. The sixth column gives the number of images used, respectively, for the estimation of magnitude and for the determination of effective wage-length.

Of special interest is a comparison of $I_\lambda$ with the color index of Seares[2] which is defined as the difference between the photographic and the photovisual magnitude

$$I_{MW}=m_{Pg}-m_{Pv}.$$

This comparison has been made in the following way. In Table II there are 77 stars for which both $I_\lambda$ and $I_{MW}$ have been deter-

[1] *Mount Wilson Contr.*, No. 97, Table IX; *Astrophysical Journal*, **41**, 206, 1915.

[2] *Loc. cit.*

TABLE II

COLORS OF THE NORTH POLAR STARS

| No. H.A. 48 | No. N.P.S. | $m_{Pg}$ | $I_\lambda$ | $p(I_\lambda)$ | $n_m \cdot n_\lambda$ |
|---|---|---|---|---|---|
| 420 | 10 | 9.48 | + .03 | 66 | 8,5 |
| 356 | | 10.37 | − .03 | 30 | 3,2 |
| 101 | 13 | 10.59 | + .21 | 173 | 7,6 |
| 386 | 14 | 10.89 | + .35 | 240 | 12,10 |
| 184 | 7r | 11.03 | + .93 | 293 | 12,10 |
| 350 | | 11.09 | + .22 | 252 | 13,11 |
| 195 | 8r | 11.10 | + .83 | 24 | 2,2 |
| 227 | 5s | 11.23 | + .90 | 300 | 14,12 |
| 197 | 15 | 11.26 | + .17 | 271 | 12,11 |
| 419 | | 11.40 | + .40 | 44 | 3,3 |
| 309 | | 11.44 | + .27 | 144 | 8,7 |
| 75 | 6s | 11.46 | + .65 | 255 | 12,11 |
| 62 | | 11.57 | + .70 | 71 | 3,3 |
| 408 | 16 | 11.63 | + .15 | 209 | 9,8 |
| 282 | 17 | 11.85 | + .61 | 76 | 4,4 |
| 430 | | 12.13 | + .71 | 57 | 3,3 |
| 67 | 9r | 12.2 | +1.28 | 6 | 1,1 |
| 320 | 18 | 12.32 | + .21 | 183 | 12,10 |
| 85 | 10r | 12.62 | + .30 | 174 | 9,9 |
| 225 | 7s | 12.66 | + .39 | 206 | 14,11 |
| 268 | 19 | 12.75 | + .25 | 194 | 10,9 |
| 494 | | 12.83 | + .66 | 107 | 6,5 |
| 57 | | 12.93 | + .43 | 124 | 9,8 |
| 302 | 20 | 12.94 | + .23 | 179 | 10,9 |
| 362 | | 13.05 | + .54 | 158 | 8,7 |
| 219 | 11r | 13.20 | +1.11 | 201 | 10,9 |
| 427 | | 13.24 | + .96 | 91 | 7,5 |
| 262 | | 13.4 | + .56 | 32 | 2,2 |
| 234 | 23 | 13.44 | + .35 | 190 | 8,8 |
| 479 | | 13.44 | + .51 | 88 | 5,5 |
| 489 | | 13.45 | + .14 | 98 | 4,4 |
| 451 | | 13.48 | + .45 | 83 | 5,5 |
| 292 | 22 | 13.52 | + .44 | 144 | 6,6 |
| 174 | | 13.54 | + .52 | 105 | 5,5 |
| 248 | 21 | 13.57 | + .79 | 144 | 6,6 |
| 389 | | 13.70 | + .60 | 116 | 5,5 |
| 151 | | 13.79 | +1.01 | 194 | 7,7 |
| 103 | | 13.8 | + .81 | 58 | 1,1 |
| 430 | | 13.82 | + .39 | 59 | 4,4 |
| 172 | | 13.84 | +1.21 | 144 | 5,4 |

TABLE II—*Continued*

| No. | | $m_{Pg}$ | $I_\lambda$ | $p(I_\lambda)$ | $n_m, n_\lambda$ |
|---|---|---|---|---|---|
| H.A. 48 | N.P.S. | | | | |
| 426 | | 13.90 | + .32 | 95 | 4,4 |
| 25 | | 14.0 | +1.00 | 56 | 1,1 |
| 34 | | 14.0 | + .05 | 28 | 1,1 |
| 300 | 12r | 14.00 | +1.17 | 149 | 5,5 |
| 279 | 24 | 14.04 | + .36 | 116 | 5,5 |
| 345 | | 14.08 | +1.50 | 141 | 5,5 |
| 90 | | 14.12 | +1.23 | 129 | 5,4 |
| 127 | | 14.13 | + .66 | 168 | 6,4 |
| 243 | | 14.15 | + .32 | 52 | 2,2 |
| 378 | | 14.15 | +1.07 | 116 | 4,3 |
| 146 | | 14.17 | + .55 | 108 | 3,3 |
| 28 | | 14.3 | + .15 | 24 | 1,1 |
| 33 | | 14.3 | + .89 | 24 | 1,1 |
| 329 | | 14.3 | + .88 | 115 | 4,4 |
| 134 | | 14.33 | + .41 | 141 | 6,6 |
| 263 | 25 | 14.35 | + .30 | 117 | 6,6 |
| 319 | | 14.38 | + .62 | 26 | 4,2 |
| 424 | | 14.4 | + .39 | 48 | 2,1 |
| 131 | | 14.43 | + .72 | 62 | 3,2 |
| 281 | | 14.45 | + .38 | 80 | 4,4 |
| 332 | | 14.45 | + .56 | 76 | 4,3 |
| 403 | | 14.45 | + .22 | 67 | 2,2 |
| 142 | | 14.47 | + .57 | 86 | 3,2 |
| 152 | | 14.48 | + .67 | 162 | 6,4 |
| 176 | | 14.48 | + .36 | 140 | 6,5 |
| 230 | | 14.53 | + .78 | 90 | 3,3 |
| 401 | | 14.53 | + .96 | 67 | 3,2 |
| 256 | | 14.55 | + .75 | 142 | 4,4 |
| 191 | 8s | 14.56 | + .66 | 148 | 5,4 |
| 13 | | 14.6 | +1.12 | 19 | 1,1 |
| 14 | | 14.6 | +1.08 | 38 | 1,1 |
| 327 | | 14.6 | + .68 | 76 | 2,2 |
| 437 | | 14.6 | +1.10 | 19 | 1,1 |
| 439 | | 14.6 | + .88 | 38 | 1,1 |
| 459 | | 14.6 | +1.30 | 19 | 1,1 |
| 480 | | 14.6 | + .63 | 19 | 1,1 |
| 391 | | 14.64 | + .52 | 64 | 5,3 |
| 265 | 9s | 14.68 | + .86 | 128 | 4,4 |
| 154 | | 14.7 | + .18 | 33 | 2,2 |
| 288 | 26 | 14.73 | + .85 | 66 | 3,3 |
| 60 | | 14.8 | + .83 | 28 | 1,1 |
| 86 | | 14.8 | + .38 | 28 | 2,2 |
| 105 | | 14.8 | + .58 | 66 | 3,3 |
| 107 | | 14.8 | +1.36 | 28 | 2,2 |
| 111 | | 14.8 | + .49 | 42 | 2,2 |

TABLE II—*Continued*

| No. | | $m_{Pg}$ | $I_\lambda$ | $f(I_\lambda)$ | $n_m : n_\lambda$ |
|---|---|---|---|---|---|
| H.A. 48 | N.P.S. | | | | |
| 125 | | 14.8 | + .57 | 61 | 3,3 |
| 175 | | 14.8 | + .31 | 42 | 2,2 |
| 333 | | 14.8 | +1.06 | 28 | 1,1 |
| 336 | | 14.8 | +1.05 | 28 | 2,2 |
| 338 | | 14.8 | + .83 | 42 | 2,2 |
| 342 | | 14.8 | + .19 | 28 | 2,2 |
| 348 | | 14.8 | − .41 | 14 | 1,1 |
| 226 | | 14.85 | + .51 | 94 | 4,4 |
| 74 | | 14.9 | + .90 | 28 | 2,2 |
| 324 | 27 | 14.97 | + .80 | 56 | 3,3 |
| 192 | | 14.98 | + .37 | 56 | 4,4 |
| 88 | | 15.0 | + .66 | 23 | 2,2 |
| 130 | | 15.0 | + .65 | 51 | 3,3 |
| 140 | | 15.0 | +1.06 | 65 | 3,3 |
| 283 | | 15.0 | +1.28 | 23 | 2,2 |
| 311 | | 15.0 | + .14 | 37 | 2,2 |
| 159 | | 15.08 | + .43 | 65 | 4,4 |
| 158 | | 15.12 | + .63 | 55 | 4,4 |
| 190 | 28 | 15.15 | + .75 | 50 | 4,4 |
| 334 | | 15.15 | + .80 | 18 | 2,2 |
| 132 | | 15.2 | +1.25 | 18 | 1,1 |
| 306 | | 15.2 | + .53 | 18 | 2,2 |
| 341 | | 15.2 | + .91 | 27 | 2,2 |
| 323 | 10s | 15.2 | + .56 | 36 | 3,3 |
| 196 | | 15.30 | + .65 | 36 | 4,4 |
| 139 | | 15.35 | +1.72 | 13 | 2,2 |
| 193 | | 15.35 | − .04 | 17 | 2,2 |
| 277 | | 15.35 | + .62 | 13 | 2,2 |
| 254 | 11s | 15.38 | + .65 | 35 | 4,4 |
| 215 | | 15.40 | +1.50 | 44 | 4,4 |
| 126 | | 15.43 | +1.04 | 8 | 3,2 |
| 165 | | 15.45 | + .43 | 13 | 2,2 |
| 163 | | 15.5 | + .76 | 4 | 1,1 |
| 201 | | 15.5 | + .19 | 11 | 2,2 |
| 205 | | 15.5 | + .76 | 8 | 1,1 |
| 218 | | 15.5 | +1.12 | 12 | 2,2 |
| 358 | | 15.5 | +1.09 | 9 | 2,1 |
| 339 | 13s | 15.6 | + .81 | 6 | 3,2 |
| 365 | 12s | 15.65 | + .94 | 10 | 2,2 |
| 295 | | 15.7 | + .28 | 4 | 2,1 |
| 164 | | 15.90 | +1.25 | 2 | 4,1 |
| 308 | | 15.9 | +1.28 | 2 | 1,1 |
| 212 | 29 | 15.95 | + .21 | 2 | 2,1 |
| 186 | | 16.1 | + .57 | 2 | 1,1 |

mined. Of these the star 439, for which the photographic magnitude of Seares and consequently $I_{MW}$ is uncertain, was omitted. Among the other 76 stars there are 40 for which the weight of $I_\lambda$ is less than 90. These and the star 101 have been combined into means of 2 or more stars as follows: 420 and 101 (combined weight of $I_\lambda$ 239); 195 and 282 (100), 319, 424, and 403 (141); 142 and 401 (153); 391 and 154 (97); 288 and 125 (127); 336, 338, and 342 (98); 324 and 130 (107); 283, 311, and 159 (125); 158 and 190 (105); 334, 341, 323, and 139 (94); 254, 215, 126, 163, 205, 218, 358, 339, 365, 295, 164, 368, 212, and 186 (148). Thus 35 single and 12 combined objects were available. These 47 objects are

TABLE III

| $m_m$ | $I_{MW}$ | $I_\lambda$ | $I_\lambda$ O−C | $m_m$ | $I_{MW}$ | $I_\lambda$ | $I_\lambda$ O−C |
|---|---|---|---|---|---|---|---|
| 10.08 | + .13 | + .16 | + 9 | 13.63 | + .60 | + .36 | − 7 |
| 10.76 | + .44 | + .35 | + 2 | 12.90 | +1.78 | +1.50 | 0 |
| 10.42 | +1.04 | + .93 | + 5 | 13.83 | +1.04 | + .66 | −17 |
| 10.82 | + .37 | + .22 | − 5 | 13.44 | +1.40 | +1.07 | − 8 |
| 10.58 | +1.03 | + .90 | + 3 | 14.04 | + .96 | + .55 | −20 |
| 11.06 | + .33 | + .17 | − 6 | 13.83 | + .98 | + .88 | +11 |
| 11.06 | + .65 | + .65 | +13 | 13.98 | + .89 | + .41 | −28 |
| 11.42 | + .39 | + .15 | −13 | 13.84 | + .48 | + .30 | − 3 |
| 11.42 | + .69 | + .66 | +11 | 14.09 | + .82 | + .35 | −28 |
| 12.08 | + .38 | + .21 | − 5 | 14.16 | + .80 | + .67 | + 6 |
| 12.36 | + .51 | + .39 | + 2 | 14.12 | + .90 | + .74 | + 4 |
| 12.48 | + .41 | + .25 | − 3 | 14.26 | + .81 | + .75 | +14 |
| 12.76 | + .51 | + .23 | −14 | 14.14 | + .71 | + .66 | +13 |
| 12.72 | + .82 | + .54 | −11 | 14.38 | + .72 | + .40 | −13 |
| 12.64 | +1.16 | +1.11 | +16 | 14.30 | + .90 | + .86 | +17 |
| 12.67 | +1.20 | + .96 | − 3 | 14.45 | + .84 | + .72 | + 8 |
| 13.35 | + .50 | + .35 | 0 | 14.38 | + .89 | + .71 | + 3 |
| 13.12 | + .63 | + .44 | − 3 | 14.58 | + .70 | + .51 | 0 |
| 12.90 | + .85 | + .79 | +12 | 14.68 | + .81 | + .73 | +12 |
| 13.20 | + .74 | + .60 | + 3 | 14.68 | + .81 | + .50 | −11 |
| 13.15 | +1.14 | +1.01 | + 8 | 14.87 | + .62 | + .69 | +26 |
| 13.04 | +1.61 | +1.21 | −14 | 14.74 | +1.00 | + .87 | + 9 |
| 13.58 | + .50 | + .32 | − 3 | 14.86 | +1.21 | +1.02 | + 6 |
| 13.15 | +1.38 | +1.17 | + 3 | | | | |

given in Table III, arranged according to the photographic magnitudes of Table II. The first column of Table III contains the mean of the photovisual and photographic magnitudes of Seares, $m_m$, which is independent of their difference $I_{MW}$. The material

of Table III was treated according to least squares with the following result:

$$I_\lambda - 0.632 = 0.895(I_{MW} - 0.810) - 0.015(m_m - 13.128)$$
$$\pm 0.054 \qquad \pm 0.014 \text{ (m.e.)}$$

or written in another way

$$I_\lambda - 0.632 = 0.888(m_{Pg} - 13.533) - 0.902(m_{Pv} - 12.723).$$

The differences O−C between the observed and calculated values of $I_\lambda$ are given in the last column of Table III. These differences do not show any distinct run, the number of changes in sign being 25, where we should expect $23 \pm 3.4$ (m.e.). The linear formulae therefore represent the observations sufficiently well.

From the first of these formulae it is seen that there is no sensible magnitude equation between $I_\lambda$ and $I_{MW}$, as the coefficient of $m_m$ practically does not exceed its mean error. This result is perhaps the most satisfactory one of the present note. As described above, special care was taken to avoid a magnitude error in the values of $I_\lambda$. Although an error of this kind does not appear in the differences $m_{Pg} - m_{Pv}$ this does not prove that the scales for $m_{Pg}$ and $m_{Pv}$ are necessarily correct; but they must be parallel.

To say the least the values of $I_\lambda$ do not indicate any such inequality between the two scales of Seares.

To $m_m = 13.128$ and $I_\lambda = 0$ corresponds $I_{MW} = +0.10 \pm 0.05$ (m.e.), although we have both intended to reduce our scale of color index to the same zero point. As the small disagreement found is only about twice its mean error its reality is not beyond question.

The mean square of the difference O−C in Table III is $0.0141 = (\pm 0.12)^2$. As this square is the sum of the squares of the mean errors of $I_\lambda$ and $I_{MW}$ the accuracy indicated is about what could be expected.

Considering the individual stars, it may be observed that the star 85, for which $I_\lambda = +0.30 \pm 0.08$ (m.e.), seems to be wrongly included with the *r* series (No. 10*r*), as it does not appear to be of deep color. My photographic magnitude 12.62 gives in connection with the photovisual magnitude 12.03 of Seares: $I_{MW} = +0.41$,

corresponding to $I_\lambda = +0.29$, thus confirming the value found directly.

On September 14, 1912, a plate (No. 207) was taken with several exposures of the Pole star in order to determine the effective wave-length of its $\Sigma$ companion 18".3 distant. From six images was found $I_\lambda = +0.11$ with weight 188, or a mean error of $\pm 0.073$. This value of $I_\lambda$ corresponds to a spectrum of A0 to A5 of the Harvard classification. Recently F. C. Leonard (*Publ. Astr. Soc. of the Pacific*, **33**, 215, 1921) has found the spectrum of the companion to be of class F0.

LEIDEN
November 12, 1921

# ELECTRIC FURNACE EXPERIMENTS INVOLVING IONIZATION PHENOMENA[1]

By ARTHUR S. KING

ABSTRACT

*Furnace temperature required to obtain the subordinate series lines of the alkali metals in absorption.*—With the central plug at about 2600°C, the stronger lines of both the first and second subordinate series of K, Cs, and Rb were readily obtained, while for Na, less than 2350° was required. The lines observed for each metal are listed. The temperature of the absorbing vapor may be taken as about 400° less than that of the plug.

*Furnace absorption spectrum of iron vapor above 2800°C.*—Using lower temperatures, no absorption lines had previously been obtained longer than λ 5507, but with a plug temperature above 3200°, practically all the high temperature emission lines were obtained in absorption, up to the end of the range photographed, λ 6700.

*Furnace spectra of Ca, Sr, and Ba vapors mixed with more easily ionized elements, K and Cs.*—*Flame lines* are not affected, but the *intensity of enhanced lines* was found to be decreased for both absorption and emission spectra, when the easily ionized vapor of K (4.3 volts) or of Cs (3.8 volts) was added, the effect being very marked for Ca (6.1 volts), somewhat less for Sr (reduction to one-third) and small for Ba (5.2 volts), while adding Na (5.2 volts) to Ba vapor produced no change. Evidently the *quenching effect* depends upon the relative ease of ionization of the two vapors. Temperatures of 1800–1830° were used.

*Furnace temperatures required to obtain certain flame lines in absorption and in emission.*—The lines λ 4227 (*calcium*), λ 4607 (*strontium*), and λ 5535 (*barium*) were obtained in absorption (tungsten lamp source) but not in emission (exposure 2 hours) at temperatures of 1275°, 1200°, and 1400°C respectively. This lag of emission over absorption may be partly due to the wideness of the lines, and is not noticeable for sharp *iron* lines.

*Relation of furnace observations to Saha's theory of spectra.*—The foregoing results verify Saha's predictions that the subordinate series of the alkalies could be obtained in absorption if sufficiently hot vapor were used, and that a gas too cool to emit light should still be able to absorb the lines of the principal series; and the results with mixed gases are such as would be expected if flame and enhanced lines are associated with neutral and ionized atoms respectively, as Saha assumes.

This paper gives the results of some experiments which bear on certain features of the theory recently advanced in a series of papers by M. N. Saha,[2] concerning the part which ionization may have in the production of certain types of spectral lines. The points tested are (1) the possibility of obtaining in absorption the lines of the subordinate series of the alkali metals, and in general, the conditions necessary for any type of lines to appear in absorption, (2) the effect, on lines presumed to be due to the ionized atom, of

[1] *Contributions from the Mount Wilson Observatory*, No. 233.

[2] *Philosophical Magazine*, **40**, 472, 809, 1920; **41**, 267, 1921; *Proceedings of the Royal Society*, A, **99**, 135, 1921

mixtures with elements having lower ionization potentials, and (3) whether, under given conditions of the metallic vapor, a line appears in absorption at a lower temperature than it can be obtained in emission.

The tube-resistance furnace offers special possibilities for an investigation of this sort by reason of the close control of the temperature and pressure. Absorption spectra also are readily obtained, either by the use of a diaphragm within the tube or by passing white light through the tube from an external source.

The special means of excitation employed will be described under each heading. The furnace chamber was usually pumped out to about 8 mm of mercury. The spectrograms were all made with the bright first order of a 15-foot concave grating, giving a scale of 3.7 A per mm.

### 1. ABSORPTION SPECTRA OF SODIUM, POTASSIUM, CAESIUM, AND RUBIDIUM

In his discussion of absorption spectra,[1] Saha notes that for these elements only the lines of the principal series have been obtained in absorption, the experiments with sodium having been by Wood[2] and those with potassium, caesium, and rubidium by Bevan.[3] Saha's explanation of the failure of the subordinate series to appear is that the temperature of the absorbing vapor was not high enough. In view of the writer's recent study[4] of the production in absorption of lines belonging to various temperature classes, this explanation seemed reasonable. The subordinate series of lines certainly require, in emission spectra, a stronger excitation than the lines of the principal series, and they should appear in absorption when the temperature is high enough to furnish a fairly large number of atoms capable of emitting the subordinate series lines. However, it was worth while to test this in the specific case of the alkali metals.

*Sodium.*—A trial of the furnace with sodium, operated as usual to give emission spectra, yielded at 2250° C and without prolonged

[1] *Philosophical Magazine*, **41**, 267, 1921. [2] *Astrophysical Journal*, **29**, 97, 1909.

[3] *Proceedings of the Royal Society*, A, **83**, 421, 1910; **85**, 54, 1911.

[4] *Mt. Wilson Contr.*, No. 174; *Astrophysical Journal*, **51**, 13, 1920.

exposure the pair λλ 5683–88 of the first subordinate series, and two pairs, λλ 5149–54 and 6155–61 of the second subordinate series, in addition to the widely reversed D lines. This showed that the production of the required atomic orbits was well within the range of the furnace.

In order to obtain the absorption spectrum, a plug of graphite was placed near the middle of the furnace tube. The spectrograph was set to give, in addition to the pairs above mentioned, those at λλ 4979–83, 4748–52, 4665–68, the plate thus including three members each of the first and second subordinate series. A plug temperature of 2650°–2700° gave distinctly in absorption all six pairs. A temperature of 2350°, however, sufficed to show the three pairs of the first or diffuse series and the strongest pair of the second or sharp series at λλ 6155–61. The absence of the weaker sharp pairs was probably due to their extreme narrowness at low temperature in a vacuum source, so that the absorption lines were below the limit of photographic resolution. Aside from this feature, the sharp and diffuse series appeared in absorption with equal ease.

The temperature of the absorbing vapor was necessarily much below that of the plug, about 400° lower, according to the experiments previously reported,[1] which have since been confirmed so often that this value may be considered a constant of the special experimental arrangement employed. This means that near the end of the heated portion of the tube there is a layer of vapor sufficiently cooler than the plug to absorb radiation from the latter. The present experiments thus indicate that sodium vapor at 2000° C or higher, with a sufficiently hot background, will absorb the lines of the two subordinate series.

*Potassium.*—A trial with a plug temperature of 2700° gave in absorption the stronger lines of the subordinate series groups near λλ 5800 and 5340. These consist in each case of one pair belonging to the first and one to the second series.

*Caesium.*—The absorption spectrum with caesium chloride in the furnace was photographed from λ 5400 to λ 7000. All but the faintest of the arc lines in this region were obtained in absorption for a plug temperature of 2600°. The absorption spectrum showed

[1] *Loc. cit.*

nine lines of the first subordinate series and five weaker ones which Saunders,[1] by reason of the constant differences occurring among them, considers to belong to the second subordinate series. The two groups are as follows:

| I | | II |
|---|---|---|
| 5466 | 6011 | 5746 |
| 5503 | 6213 | 5839 |
| 5635 | 6724 | 6034 |
| 5764 | 6974 | 6355 |
| 5845 | | 6587 |

Three lines, given by Saunders as λλ 5847.8, 6217.6, 6983.8, and classed by him as satellites of adjacent strong lines, are visible also in the absorption spectrum. In the arc the lines which presumably belong to the second series are very unsymmetrical toward the red, while the absorption furnace in vacuum gives for each of these a narrow line at the extreme violet side of the position of the arc line. Dissymmetries occurring among the lines of the first series are also eliminated by the furnace. When the furnace spectrum is more completely photographed, with proper standards, a revision of the wave-lengths can be made and the character of the second series definitely settled.

*Rubidium.*—For this element also, there was no difficulty in obtaining in absorption nearly all of the lines given by the arc. In the range λ 5100 to λ 6400, the following lines of the first and second subordinate series showed as absorption lines, only the fainter members of the second series being absent:

| I | | II |
|---|---|---|
| 5151 | 5648 | 5654 |
| 5196 | 5724 | 6071 |
| 5260 | 6207 | 6160 |
| 5363 | 6299 | |
| 5432 | | |

While not bearing so directly upon the ionization theory as the foregoing, a series of experiments with the iron spectrum in absorption may be mentioned. In the previous investigation,[2] when the

[1] *Astrophysical Journal*, **20**, 188, 1904. [2] *Loc. cit.*

furnace was heated in vacuum with the plug at about 2600° C, no iron lines were obtained in absorption to the red of λ 5507, an effect probably due, as was explained then, to the high-temperature character of the iron lines in the yellow and red. Later experiments have extended this limit. The furnace, operated at atmospheric pressure, had its tube thinned in the central region to permit very high temperatures of the plug and to give a strong gradient in the portion of the tube containing the iron vapor. Plug temperatures above 3200° C, the highest reading of the pyrometer employed, were thus obtained. The spectrum was photographed to λ 6700, and gave in absorption practically all of the lines shown in emission by the furnace at high temperature.

The conclusion from these results is that a sufficiently high temperature of the absorbing vapor, and proportionately higher temperature of the incandescent background, give in absorption all lines which the furnace can emit, and experience has shown these to include probably all lines not requiring conditions similar to those of the spark discharge.

This condition, which is an obvious consequence of the difference in excitation required to produce different groups of lines in the same spectrum, is enunciated by Saha to the effect that in order to reverse a given set of lines, it is not sufficient to send a beam of white light through a cooler layer of the vapor, but that "in the atoms present, there should be a fairly large number with orbits corresponding to the first term of the pulse of radiation to be absorbed. Thus in order that lines (*2p–md*) may be absorbed (as those of the subordinate series of the alkalis), we must have a sufficient number of atoms with (*2p*) orbits."[1]

### 2. EFFECT OF MIXTURES OF ELEMENTS HAVING DIFFERENT IONIZATION POTENTIALS

In the writer's studies of electric furnace spectra, it has been noted repeatedly that a mixture of substances may be vaporized without any suppression of the spectrum of one by reason of the presence of the others. In contrast with the arc containing such a mixture, each vapor appears to give its spectrum as if it were alone.

[1] *Op. cit.*, p. 277

No evidence against this view has been found as regards the great majority of the lines emitted by the furnace. These lines, according to the modern view, are emitted by various electron orbits of the "normal atom," that is, an atom which is not ionized through the removal of one or more electrons. The furnace is capable, however, of emitting, in general rather faintly, some of the enhanced lines, which are regarded as due to the ionized atom. For these lines, the degree of ionization is a determining factor, and this may be affected by conditions other than the temperature and pressure which are controlled in the furnace.

The calcium spectrum contains various series emitted by the normal atom, and also the enhanced lines H and K arising from the ionized atom. Designating the normal atom by Ca and the ionized atom and its lost electron by $Ca^+$ and $e$, respectively, Saha[1] states, "At a given temperature and concentration, a definite equilibrium will be established between the proportions of Ca, $Ca^+$, and $e$, as represented by the van't Hoff formula of reversible chemical action, $Ca \rightarrow Ca^+ + e$."

If then we add a large supply of electrons from some substance which becomes ionized more readily than calcium, fewer electrons from the calcium can exist in the free state, and the proportion of $Ca^+$ atoms becomes less, with a corresponding weakening of the H and K lines, relative to the lines of the normal atom. Since the ionizing potential of an element may be taken as the measure of its ionization under a given stimulus, a mixture of calcium with an element having a lower ionization, such as potassium, caesium, or rubidium, should alter the calcium ionization to an appreciable extent. A fairly crucial test of the basic assumptions of Saha's theory should thus be possible, and Dr. H. N. Russell has suggested to the writer that the control of conditions in the electric furnace affords an excellent means of applying the test. The following experiments were accordingly carried out for the spectra of calcium, strontium, and barium.

*Calcium.*—A temperature of about 1800° C was selected, which gives the H and K lines distinctly, and yet with intensities low enough to register easily any disturbance in the vapor producing

[1] *Proceedings of the Royal Society*, A, **99**, 139, 1921.

them. The first experiments were with absorption spectra, which permit short exposures, during which there would be little change in the vapor within the tube. Instead of using a plugged tube, white light from a strong tungsten projection lamp was sent through the tube of the vacuum furnace. As the filament of the lamp was many hundred degrees hotter than the furnace tube, it could safely be assumed that the vapor in the highly heated portion produced the absorption, instead of that near the end as was evidently the case when a plug was used. The first mixture tried was calcium and potassium, whose ionization potentials are 6.08 and 4.32 volts, respectively. The absorption spectrum was first photographed with metallic calcium alone in the tube. The furnace was then recharged with a mixture of calcium and potassium chloride, and a second exposure made under the same conditions as the first. The quenching effect on the H and K lines was very distinct. With calcium alone, H and K appeared as absorption lines of moderate strength, comparable with some of the lines in the group of six near λ 4300 and in the series triplet λλ 4425–55. When calcium and potassium were vaporized together, scarcely a trace of H and K was to be seen, while the comparison lines due to the normal atom were practically unchanged. The nearly equal width of the strong flame line λ 4227 in the two exposures indicated that the density of calcium vapor was nearly the same in both cases. The experiment was repeated, the tube being wiped out before charging with the calcium alone, but the potassium was not entirely removed, as was shown by the presence of the violet potassium pair. H and K were thus somewhat weakened in the first exposure, but much more affected in the second with a large supply of potassium.

The comparison was repeated with a new tube and metallic potassium, instead of the chloride, for the second exposure. This eliminated a band structure in the H and K region which may have had some effect in the previous experiment. H and K were again much weakened with reference to the other calcium lines, though the narrower potassium lines showed that the metal did not give as effective a mass of vapor in the tube as was obtained with the chloride.

Experiments with emission spectra were next made, first with potassium chloride and then with caesium chloride, which were

compared with calcium alone. The exposures were much longer, with resulting variations in the supply of vapor during the runs, but the effects observed for absorption were repeated, H and K being distinctly reduced in intensity when a mixture was used. As was to be expected, the effect of caesium, with an ionization potential of 3.81, appeared greater than that of potassium, but the effective quantities of the two may have been different, and at present it is desired to lay stress only on the general effect of a substance with lower ionizing potential.

*Strontium.*—The experiments with strontium were made first for the absorption spectrum with the chloride in the tube, at a temperature of 1860° C, then with a mixture of strontium chloride and potassium chloride. The enhanced lines λλ 4078 and 4216 were compared with the flame line λ 4607 and a group of strong furnace lines from λ 4722 to λ 4876, presumably due to the normal atom. As with calcium, the enhanced lines showed a decided effect from the mixture of potassium. λ 4078 was disturbed by a band spectrum, but λ 4216 was favorable for comparison, and relatively to the other lines, showed a reduction to not more than one-third of its former intensity. A repetition of the experiment, this time for the emission spectrum at 1830°, permitted both λ 4078 and λ 4216 to be observed. They were found to be weakened by the addition of potassium to approximately the same degree as in absorption. This tube was afterward used for the experiment with calcium alone and calcium with caesium in emission, mentioned in the preceding section. The stronger strontium lines persisted and the enhanced pair was much reduced by the addition of caesium.

*Barium.*—A series of experiments was made for the absorption spectrum of barium at λ 1830° C, using both barium chloride alone and also mixtures of either potassium chloride or caesium chloride. The enhanced lines λλ 4554 and 4934 were compared with λ 5535 and the strong furnace lines from λ 5778 to λ 6063. The effect on the enhanced lines was much smaller than with the corresponding lines of calcium and strontium, but a perceptible relative weakening of the enhanced pair followed the addition of both of the substances with lower ionizing potentials.

The ionization potential of barium is lower than that of strontium or calcium, being in fact halfway between potassium and calcium. This and the fact that the region occupied by the barium lines is not so favorable for the observation of absorption effects as the portion farther to the violet may be responsible in some measure for the smaller influence in the case of barium. A mixture with sodium vapor, whose ionization potential is almost exactly the same as that of barium, was tried and no effect whatever could be detected.

### 3. TEMPERATURES REQUIRED FOR THE INITIAL APPEARANCE OF A GIVEN LINE IN ABSORPTION AND IN EMISSION

It is desirable to extend the observations on this point much farther than has yet been possible, but the tests made on lines of a favorable type have given definite results, which will be reported here.

In his discussion of the radiation of the characteristic lines of the neutral atom,[1] Saha reasons that a mass of gas too cool to emit any light should still be able to absorb lines of the principal series. Further heating causes the atoms to progress toward ionization, and radiation will follow as a result of the mutual interchange of orbits. When the gas can emit the principal series rather strongly, it should absorb the lines of the subordinate series, a still higher temperature producing the latter series in emission.

The tests thus far made have been concerned with the question whether there is a temperature at which the principal series Ca $\lambda$ 4227, Sr $\lambda$ 4607, and Ba $\lambda$ 5535 will appear in absorption and not in emission. The higher critical stage at which these lines are emitted and at which the subordinate series may show only in absorption has not yet been looked for.

The experimental arrangement provided a transition from absorption to emission conditions by simply turning off a 900-watt tungsten lamp, whose light was directed through the tube during the absorption exposure.

The exposures required for the absorption and the emission spectrograms were very different. For absorption one-half to one

[1] *Proceedings of the Royal Society*, A, **99**, 140, 1921.

minute was sufficient to produce a continuous ground of the proper strength to show any absorption which was present. For the emission spectrum, it was a question of registering a very faint image, and a period of two hours was considered a fair time, in view of the power of the spectrograph, to show anything which the vapor could emit. Absorption spectra were photographed at the beginning and end of the emission run.

At 1275° C, λ 4227 of calcium appeared in absorption, 0.5 A in width, persisting as a well-defined absorption line at the close of a two-hour exposure for the emission spectrum, but the line failed to show in emission. At the same temperature, a very faint trace of λ 4607 of strontium could be seen as an emission line, but nothing whatever appeared in emission at 1200°, though at the latter temperature the absorption line λ 4607 was strong at the beginning and end of the period.

The barium line λ 5535 could be seen very faintly in emission at 1500°; but with the temperature held close to 1400°, at no time rising above 1440°, the line entirely disappeared. The absorption line was still strong at 1400°.

Some inherent difficulties must be recognized in tests of this sort. Lines such as the three just considered have, when plenty of the material is present, low photographic density and a considerable width, which features should make them appear in absorption under minimum excitation. The same characteristics retard their development through the threshold condition in emission. Lines which tend, when faint, to become very narrow, as is the case with the great bulk of furnace lines, hold up relatively better in emission than in absorption as the temperature becomes lower; since in absorption the very narrow line may disappear too soon by reason of failure of the spectrograph to produce the resolution of the continuous spectrum which enables an absorption line to be perceived. This fact, which with the narrow lines of the vacuum furnace would make the emission line in its early stages easier to see than the absorption line, may account for the fact that in the iron spectrum, emission and absorption spectra for the same temperature of the furnace are very nearly duplicates. While the series for iron are not known, the lines fall into definite temperature

classes,[1] and the initial appearance of a line of certain type may be watched. An iron line in this sensitive state and with proportional exposures, shows as readily in emission at a given temperature as in absorption. This was tested through a range extending from the ultra-violet into the red. From the green into the red, it is somewhat easier, at a certain temperature, to be sure of the presence of a faint iron line in emission than in absorption. We may therefore say that the evidence thus far does not always favor the absorption method as a means of producing more numerous lines than are shown under equal excitation in emission, though the earlier appearance in absorption of principal series lines appears to take place in the spectra of calcium, strontium, and barium.

MOUNT WILSON OBSERVATORY
January 1922

[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

# CARBON DIOXIDE ABSORPTION IN THE NEAR INFRA-RED

By E. F. BARKER

ABSTRACT

*Infra-red absorption bands of $CO_2$ at 2.7 and 4.3 $\mu$.—New absorption curves* have been obtained, using a special prism-grating double spectrometer of higher resolution (Figs. 1–3). The 2.7 $\mu$ region, heretofore considered to be a doublet, proves to be a pair of *doublets*, with centers at approximately 2.694 $\mu$ and 2.767 $\mu$. The 4.3 $\mu$ band appears as a single doublet with center at 4.253 $\mu$. The frequency difference between maxima is nearly the same for each of the three doublets, and equal to $4.5\times10^{11}$. Complete resolution of the band series was not effected, even though the slit included only 12 A for the 2.7 $\mu$ region, but there is evidently a complicated structure, with a "head" in each case on the side of shorter wave-lengths. The existence of this head for the 4.3 $\mu$ band is also indicated by a *comparison with the emission spectrum* from a bunsen flame, and the difference in wave-length of the maxima of emission and absorption is explained as a temperature effect similar to that observed with other doublets.

*Molecular structure of $CO_2$.*—If a linear configuration of the three atoms is assumed, the *moment of inertia* of the molecule, computed from the doublet frequency difference, comes out $50\times10^{-40}$. This gives the distance from the central C atom to each O atom as $0.97\times10^{-8}$ cm. Since the linear molecule could have only three modes of vibration, it is suggested that the two doublets at 2.7 $\mu$ may correspond to two successive changes in stationary state for the same vibration.

*Echelette gratings*, used in the research, were ruled with 3000, 1000, and 500 lines per inch, on surfaces of aluminum.

In the prismatic spectrum of carbon dioxide three infra-red absorption bands have been observed,[1] at approximately 2.7 $\mu$, 4.3 $\mu$, and 14.7 $\mu$. The first region shows two absorption maxima with no indication of fine structure, but, from its behavior when the gas is subjected to various temperatures and pressures, von Bahr concluded that it is not a simple doublet. The region at 4.3 $\mu$ is narrow and for it no structure was observed. Using small echelette gratings, Trowbridge and Wood[2] found several maxima and minima in the emission spectrum from a bunsen flame between 4 $\mu$ and 4.5 $\mu$, but they apparently did not take into account the reversal effect due to atmospheric absorption. These two regions have now been re-examined with the maximum available dispersion, using a prism-grating spectrometer designed for the purpose.

[1] Von Bahr, *D. Phys. Ges.*, **15**, 710 and 1150, 1913; Burmeister, *D. Phys. Ges.*, **15**, 589, 1913.

[2] *Philosophical Magazine*, **20**, 898, 1910.

The arrangement of the optical system is shown in Figure 1. The prism is of rock salt, with an angle of 38° and faces about 7×10 cm. It separates the incident beam into a spectrum which is focused at the plane of a slit forming the source for the grating spectrometer; thus a narrow spectral region is presented to the grating for analysis, and overlapping of spectra is avoided. The detecting system is a silver-bismuth thermopile actuating a high sensitivity galvanometer of the Paschen type. An absorption cell with windows of thin glass or mica is placed immediately behind the first slit, and may be moved into and out of the optical path.

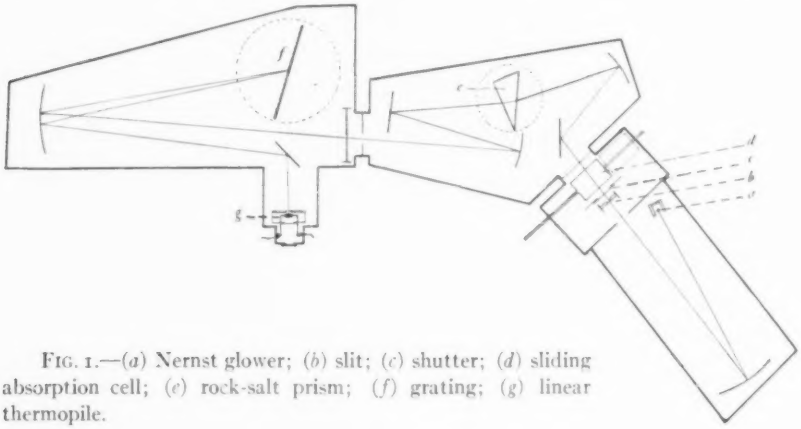

FIG. 1.—(*a*) Nernst glower; (*b*) slit; (*c*) shutter; (*d*) sliding absorption cell; (*e*) rock-salt prism; (*f*) grating; (*g*) linear thermopile.

The whole optical system is inclosed in a heavy metal case supplied with valves so that it may be thoroughly washed, dried, or even evacuated, as desired.

The region of 2.7 $\mu$ is accessible with a grating having approximately 15,000 lines per inch, and has also been examined in the first and second orders of a 5000-line grating. For the 4.3 $\mu$ band the latter grating failed to give spectra of measurable intensity and it has been necessary to prepare a number of coarser ones having approximately 3000, 1000, and 500 lines per inch. These have been ruled in the echelette form on 5″×7″ aluminium surfaces, and this department is indebted to the department of physics of Johns Hopkins University for the use of one of the Rowland ruling machines.

*Observations.*—Three absorption curves for the 2.7 $\mu$ band have been mapped with different dispersions, corresponding to approximately 55, 25, and 12 angstrom units, included by the half-millimeter slit. The absorption cell was five centimeters in length with glass windows about 0.25 mm thick. These extended beyond the cell so as to be traversed by the radiation even when the cell itself was removed from the optical path. Mica windows were used for greater wave-lengths, with a similar arrangement for compensation, but were avoided here on account of the absorption band of mica near 2.7 $\mu$. The observations consisted in noting ten or twelve galvanometer deflections for a given setting of the grating, with the cell in and out alternately, from which the mean relative decrease in deflection due to the $CO_2$ absorption was computed. The interval between successive grating positions was somewhat less than the slit width.

The three different dispersions all show this absorption region to consist of a pair of doublets, so that we have, not a single typical band, but two, lying close together, with centers at approximately 2.69 $\mu$ and 2.76 $\mu$. The use of different gratings, and the method of observing alternately with and without the absorption cell, completely eliminate the possibility of attributing this result to peculiarities of the ruled surface.

The curves of Figure 2 show the percentages of absorption obtained in the second-order spectrum of the 5000-line grating and the first-order spectrum of the 15,000-line grating. The existence of a complicated structure in these bands is very evident, but the available resolution is not sufficient to separate the individual members of the series. When using the 15,000-line grating the angle between the normal and the incident beam was 54°, so that obviously little is to be hoped for from finer gratings. The half-millimeter slit might be narrowed somewhat if a more intense source or a more sensitive detecting device were available.

The absorption band at 4.3 $\mu$ is narrow and intense, and may be observed as an atmospheric absorption even after long-continued washing of the spectrometer with $CO_2$—free air. That it is actually attributable to carbon dioxide is, however, established by the measurements with a 3 cm cell containing pure, dry $CO_2$, which

absorbs more than 90 per cent of the incident energy throughout a considerable range of frequencies. With both the 1000-line and the 3000-line gratings the band appears as a distinct doublet, though there is little indication of the serrated structure which might be anticipated with higher dispersion. Figure 3A shows the atmospheric absorption obtained in the second-order spectrum from

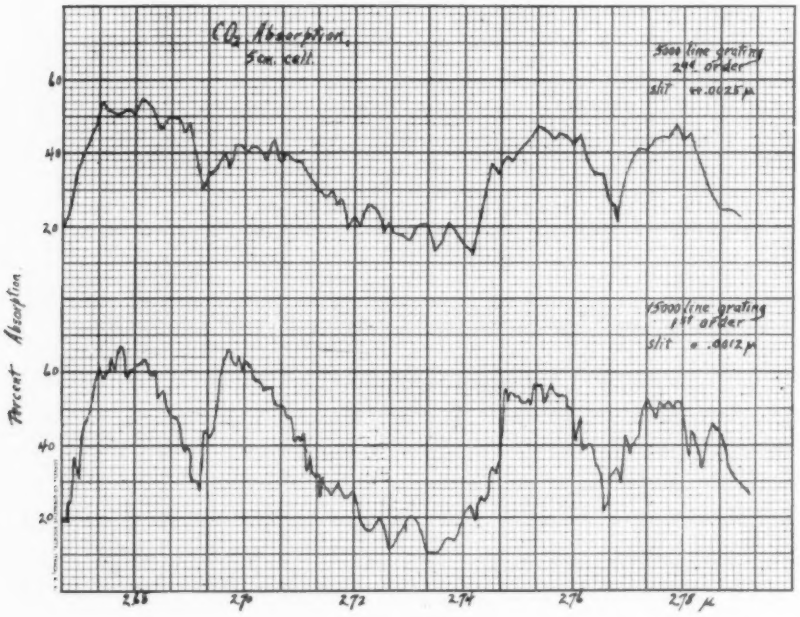


FIG. 2

the 1000-line grating, the slit including 150 angstrom units. Figure 3B indicates the method employed to compute the percentage absorption, and Figure 3C is the absorption curve obtained in the first order of the 3000-line grating. The effect of introducing the absorption cell is shown in each. The dotted line of Figure 3B indicates the probable course of the energy curve for zero absorption, which could only be observed at the limits of the band. Its inclination to the horizontal is partly due to the falling off of energy as wave-lengths increase, and partly to the chosen prism setting.

The correction for slit width and slope of the energy curve[1] very slightly accentuates the doublet form, but does not displace the center nor the maxima appreciably.

The great intensity of the atmospheric absorption at 4.3 $\mu$ would seem to indicate that the emission band from a bunsen flame should be rapidly absorbed, though experimentally energy of approximately this wave-length is easily obtained through thick layers of

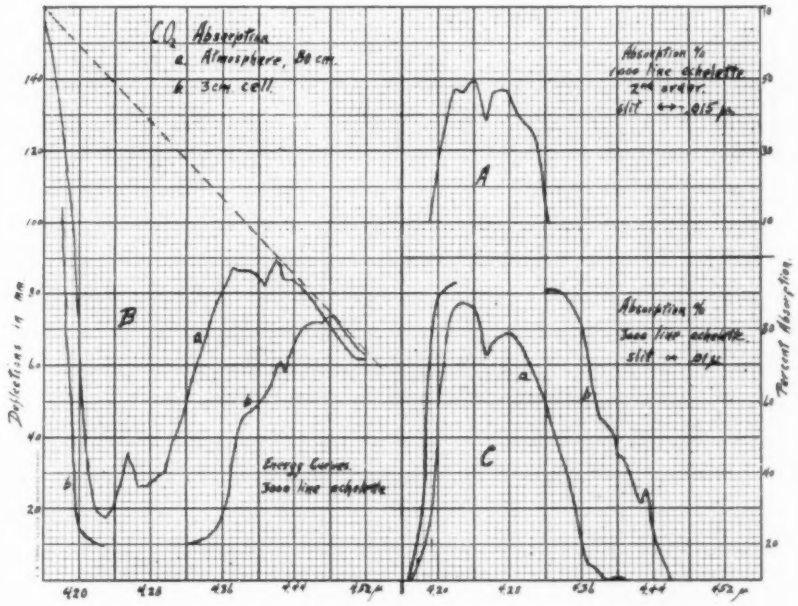


FIG. 3

air. The explanation is immediately obvious from Figure 4, which shows on the same scale the absorption of cold atmospheric $CO_2$ and the emission from the hot flame. As has been observed for other doublets, the widening due to increased temperature occurs almost entirely on the side of longer wave-lengths. From these curves it is evident that the doublet form would not be expected in the emission band unless precautions are taken to effectively eliminate absorption by cooler layers of $CO_2$—except, of course,

[1] Paschen, *Annalen der Physik*, **60**, 712, 1897.

that the "reversal" reveals an inverted doublet produced by absorption, leaving three apparent maxima in the energy curve as observed. Of these, the weakest, corresponding to the central notch in the absorption curve, occurs at 4.25 $\mu$, while the positions of the others, which we observe at approximately 4.2 $\mu$ and 4.4 $\mu$,

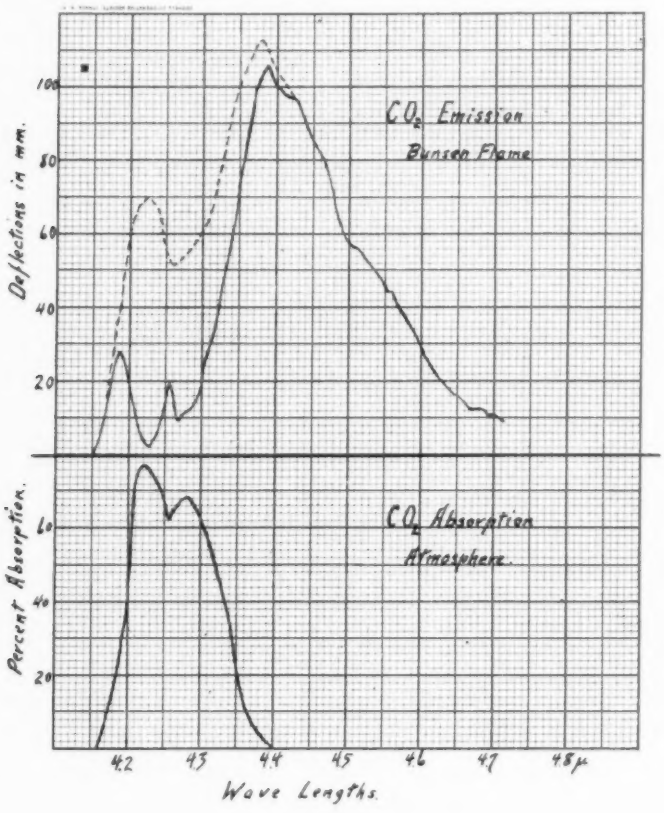


FIG. 4

depend upon the mass of gas in the cooler absorbing layers between the flame and the thermopile. Trowbridge and Wood (*loc. cit.*) observed maxima at 4.2 $\mu$, 4.4 $\mu$, and 4.5 $\mu$ with one grating, and at 4.32 $\mu$ and 4.43 $\mu$ with another. The dotted line in Figure 4 shows approximately what the energy curve would have been had no absorption occurred.

Although from the measurements here presented it is not possible to isolate the various band series, several interesting conclusions may nevertheless be drawn. In the doublet of Figure 3 and both those of Figure 2 the side toward shorter wave-lengths is steep while the bands appear broadened toward longer wave-lengths; also the higher frequency maximum is the more intense. This suggests band series similar to those of the halogen acids[1] in which the individual members are crowded on the side of shorter wave-lengths and separated by increasing intervals on the other side. For the 4.3 $\mu$ band it would appear that our measurements approach very close to the head of the series, since increases in the mass of the absorbing gas and in the temperature (Figures 3C and 4) build up the absorption almost entirely on one side.

A simple band series like that of HCl would be expected only if the $CO_2$ molecule were linear, as in Langmuir's model, with a single effective moment of inertia. The spacing of members near the middle of the band would then be about $h/4\pi^2 I$. A fair approximation to the value of $I$ may be obtained from the frequency differences of the two maxima in the various doublets. Calling this interval $2\nu_m$, we have, since $\nu_m$ is the most probable frequency of rotation,

$$I = \frac{RT}{4\pi^2 N \nu_m^2} = \frac{1.11 \times 10^{-15}}{\nu_m^2}.$$

Table I shows the values of $\nu_m$ taken from Figures 2 and 3, with corresponding values of $I$.

TABLE I

| Band Center | Maxima | $\Delta\lambda$ | $\nu_m$ | $I$ |
|---|---|---|---|---|
| 2.694 | 2.677, 2.698 | .021 | $4.35 \times 10^{11}$ | $53.4 \times 10^{-40}$ |
| 2.767 | 2.753, 2.776 | .023 | 4.52 | 49.5 |
| 4.253 | 4.225, 4.280 | .055 | 4.56 | 48.6 |
| Mean | | | | 50.5 |

For a symmetrical linear molecule with moment of inertia equal to $50.5 \times 10^{-40}$, each O atom would lie at a distance of $0.975 \times 10^{-8}$ cm from the central $C$ atom, a not improbable dimension.

[1] Imes, *Astrophysical Journal*, **50**, 251, 1919, and Colby and Meyer, *Astrophysical Journal*, **53**, 300, 1921.

This value of $I$ is, however, much larger than that derived from the thermal constants of the gas, assuming three principal moments of inertia, $A$, $B$, and $C$. Thus Schames[1] computes for carbon dioxide

$$\sqrt[3]{ABC} = 5.80 \times 10^{-40}.$$

Bjerrum[2] has discussed the triangular $CO_2$ molecule, deriving three possible vibration frequencies. From the laws of motion for the linear form it is easily shown that there are two degrees of freedom for vibration along the axis, to which we may add one vibration frequency for motion of the atoms at right angle to the axis. Thus either assumption apparently predicts three fundamental vibration spectra. Our analysis, however, reveals four bands, none of which may be considered as a so-called harmonic. The close proximity of the two bands near 2.7 $\mu$ suggests as a possible explanation that these may both be "fundamentals" of the same vibration, one corresponding to the vibrational transition from state 0 to state 1, the other to the transition from state 1 to state 2. Such a double fundamental would be expected if the motion is not simple harmonic, and has been predicted for HCl at 3.4 $\mu$ with a frequency difference for the two bands about the same as observed here.[3] This explanation would, however, involve the probable existence of a harmonic which has never been observed at about 1.4 $\mu$, a region in which water vapor has an intense absorption band. The almost equal intensities of the two series of Figure 2 would indicate a number of molecules in the first state of vibration almost equal to the number in the zero state at ordinary temperatures. The ultimate decision as to whether or not the arrangement of atoms in the carbon dioxide molecule is linear or triangular must await an analysis of the bands with sufficiently high resolution to reveal the series of which they are constituted.

This investigation has been carried on at the Physical Laboratory of the University of Michigan under a Research Fellowship of the National Research Council.

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[1] *Physikalische Zeitschrift*, **21**, 38, 1920.

[2] *D. Phys. Ges.*, **16**, 737, 1914.

[3] Kratzer, *Zeitschrift für Physik*, **3**, 289, 1920.

# THE BROADENING OF THE BALMER LINES OF HYDROGEN WITH PRESSURE

By E. O. HULBURT

ABSTRACT

*Photographic width of the Balmer lines of hydrogen for pressures up to 135 mm.*—There is some question as to whether among the possible causes of the widening of lines with pressure, the decrease of mean free path is important. A theoretical calculation of the width to be expected for hydrogen lines if the length of the train of waves emitted is limited by the time between molecular collisions, gives for a temperature of 600° C. and a pressure of 100 mm a width of about 0.04 A, between wave-lengths where the intensity is one-tenth of that at the middle. Experimentally, however, it was found that a pressure of about 100 mm gave photographic images 80 A wide. Purified hydrogen in a capillary discharge tube was excited by condensed discharges and a series of spectrograms were taken for pressures from 0.2 to 135 mm. The exposure time was 30 minutes in each case and the discharge was regulated so that the intensity of the central portion of Hβ was practically the same for all pressures. As shown by Figure 2, Hβ, Hγ, and Hδ behave very much alike. The fact that the actual width is much greater than that computed theoretically indicates that the widening of spectrum lines is probably due to some other cause, perhaps electrical fields.

*Introductory.*—The former Lord Rayleigh has summarized the causes which interfere with the absolute homogeneity of spectrum lines under five headings:[1] (1) The translatory motion of the radiating particles in the line of sight operating in accordance with Doppler's principle. (2) A possible effect of the rotation of the particles. (3) Disturbance depending on collision with other particles either of the same or of another kind. (4) Gradual dying down of the luminous vibrations as energy is radiated away. (5) Complications arising from the multiplicity of sources in the line of sight. To which is added a sixth cause proposed and investigated by Merton.[2] (6) The effect of an electric field on the radiating particle. The electric field may be that imposed on the electrodes of the tube or may be the field of neighboring atoms as suggested by Stark,[3] or a superposition of the two.

The researches of Fabry and Buisson[4] and Michelson[5] have demonstrated that for pressures of the luminous gas below one

[1] *Philosophical Magazine*, **29**, 274, 1915.

[2] *Proceedings of Royal Society*, **92**, 322, 1915.

[3] *Elektrische Spectralanalyse chemischer Atome*, 1914.

[4] *Journal de Physique*, **2**, 442, 1912.

[5] *Astrophysical Journal*, **2**, 251, 1895.

millimeter of mercury (1) is probably the only cause of broadening of the spectrum line. This cause is, however, impotent to explain the tremendously greater broadening produced in certain lines by the use of either greater pressures or condensed discharges, and therefore recourse must be had to the other causes mentioned. Of these the effects of (2) and (4) must perforce be omitted from discussion, for our present knowledge of atomic constitution and the nature of radiation is insufficient for a successful consideration of the effect of rotation of the luminous particle and of the damping of the vibration as the energy is radiated away. We also neglect (5), for this merely aggravates the effects of the other causes. There remain (3) and (6). From their recent work on the energy distribution in the broadened lines of hydrogen, helium, and lithium by high pressures stimulated by condensed discharges, Merton and Nicholson[1] concluded that cause (6) was all important in producing the broadening and that (2) played no part whatsoever. Their observations were concerned with the *form* of the broadened line and did not touch upon the *magnitude* of the broadening. In view of the novelty of this conclusion it seemed of interest to determine the magnitude of the broadening. The present work was carried out to this end and the photographic widths of $H\beta$, $H\gamma$, and $H\delta$ of the Balmer series of hydrogen were measured for pressures from 0.2 to 135 mm of mercury. The broadening which might be expected from cause (3) under the conditions of the present experiment was then calculated and was found to be considerably less than the observed broadening. This evidence thus supported the conclusions of Nicholson and Merton in that it threw doubt on the validity of (3) as a cause of the widening.

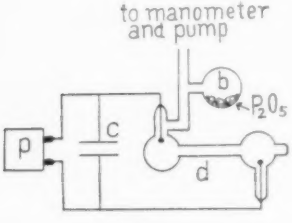
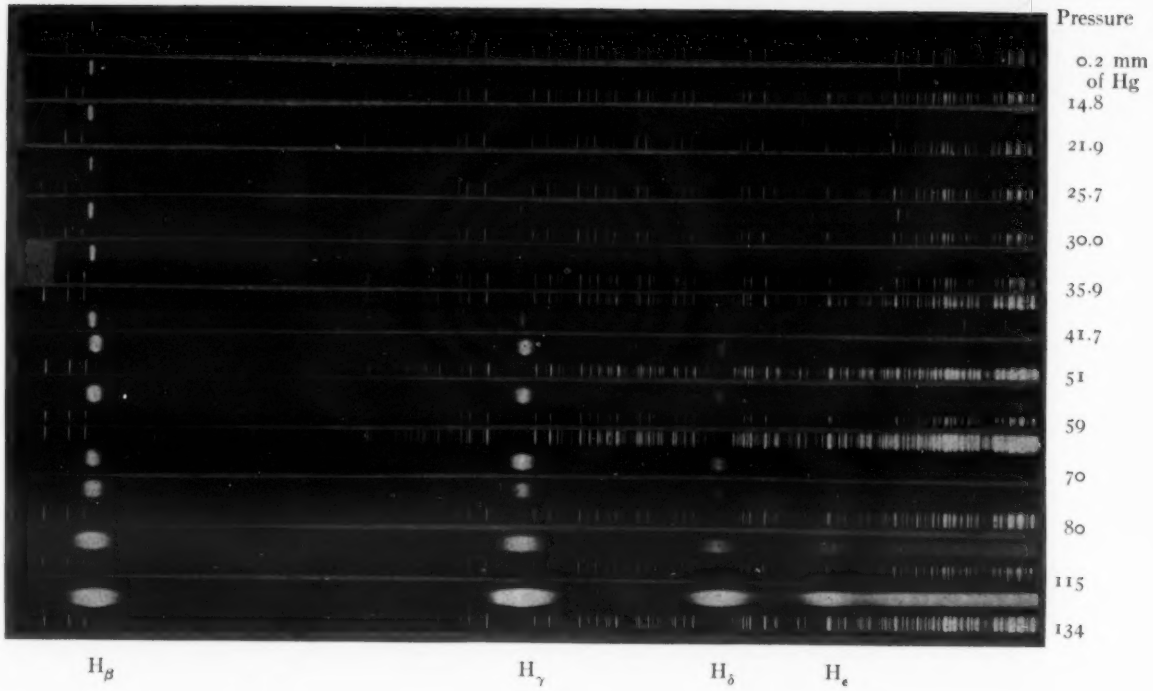



FIG. 1

*Experimental.*—Hydrogen, prepared from hydrochloric acid and zinc and purified through sodium hydroxide solution and concentrated sulphuric acid, was passed into the end-on discharge tube *d*, Figure 1, made of glass with aluminum electrodes arranged out of line with the

[1] *Philosophical Transactions*, Vol. 216, 459, 1916.

## PLATE VII


Pressure
0.2 mm of Hg
14.8
21.9
25.7
30.0
35.9
41.7
51
59
70
80
115
134
$H_\beta$
$H_\gamma$
$H_\delta$
$H_\epsilon$

capillary. The capillary was 10 cm long and 5 mm in diameter. A bulb *b* with phosphorous pentoxide was sealed to the tube. The tube was connected by glass tubing about a meter in length to a bulb of perhaps 200 c.c. capacity and a mercury manometer. The total capacity of the discharge tube and glass system, perhaps 300 c.c., was filled with hydrogen at various pressures and no change in pressure was observed when the discharge was run continuously for an hour. The condenser *c*, Figure 1, of capacity of about 0.01 $\mu$ F, which discharged through the tube, was charged by the 25,000 volt transformer *p*. The spectra were photographed in the first order of a concave grating of 2 meters radius of curvature and ruled 14,435 lines to the inch, the dispersion being about 8.8 A per mm.

Spectra of hydrogen were taken for a series of pressures from 0.2 to 135 mm. These are shown in Plate VII with the iron arc comparison spectrum. The tube was maintained rigidly in position throughout the series and the time of exposure was 30 minutes for each spectrogram. In order to obtain conditions for the various pressures that might be compared the procedure was as follows: A glass prism spectroscope was trained on the hydrogen tube so that H$\beta$ could be observed visually at the same time that the spectrum was being photographed. A small region at the center of the line was isolated by the second slit of the prism-spectroscope and observed by a low-power eyepiece. A comparison line was reflected into the field of view of the eyepiece obtained by illuminating a portion of the first slit with the light from a tungsten lamp filtered through a greenish-blue filter. The intensity of the comparison line was maintained constant, and by adjusting the power consumed in the hydrogen tube the intensity of the central portion of H$\beta$ was kept approximately constant for each pressure. It is worthy of remark that the condensed discharge through the hydrogen tube was at all times an oscillatory one, the frequency as measured by a wave-meter being $0.28 \times 10^6$ alternations a second. For this reason the electrical power consumed in the tube could not be determined in a simple manner by electrical methods.

The widths in angstrom units of H$\beta$, H$\gamma$, and H$\delta$ were measured from the photographs of Plate VII, and were plotted in Figure 2

as ordinates against pressures in mm of mercury as abscissae. The measurement of the width was made simply by judging the points on the edges of the lines on the plates where the blackening became slight and measuring between these points. This was manifestly inaccurate but was deemed sufficiently precise for the purpose in hand. The last two points on each of the curves were too high, for at these two highest pressures it was not possible to

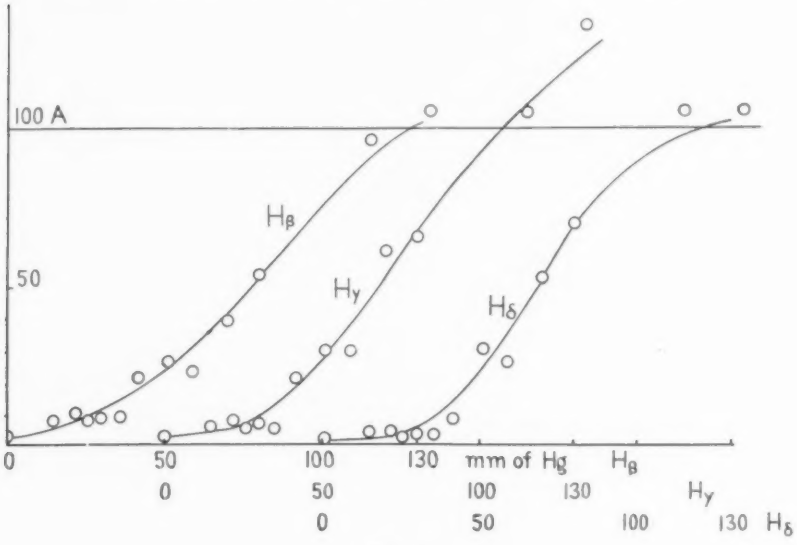



FIG. 2

reduce the intensity of the center of H$\beta$ to the intensity used at the lower pressures and still maintain sufficient voltage to render the gas luminous.

*Theoretical.*—The third cause of widening mentioned in the introduction is the disturbance of the free vibration of the radiating particle due to encounters with other bodies. The simplest supposition open to us is that an entirely fresh start is made at each collision, so that we have to deal with a series of regular vibrations of finite length. The problem thus arising has been treated by Godfrey[1] and Schönrock.[2] The Fourier analysis of the limited

[1] *Philosophical Transactions*, **195**, 329, 1899.

[2] *Annalen der Physik*, **22**, 209, 1907.

train of waves of length $r$ gives for the intensity of various parts of the spectrum line

$$I = I_0 \frac{\sin^2 \pi r\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)^2} \tag{1}$$

where $I_0$ and $\lambda_0$ are the intensity and wave-length respectively of the center of the spectrum line and $I$ and $\lambda$ are the respective quantities in any part of the line. $r$ is the length of the train of waves in cms. $\lambda_0$ is the wave-length which would be dominant if $r$ were infinitely long.

We calculate $r$ from the Kinetic Theory of Gases,[1] to be

$$r = \frac{c}{4\nu\sigma^2}\sqrt{\frac{m}{\pi RT}} \tag{2}$$

where $\nu$ is the number of molecules per unit volume at the pressure and temperature in question, $\sigma$ is the diameter of the sphere of molecular action from the Kinetic Theory standpoint, $m$ is the mass of the molecule, $c$ is the velocity of light *in vacuo* and R is the gas constant.

Substituting (2) in (1) gives

$$I = I_0 \frac{\sin^2 \frac{\pi c}{4\nu\sigma^2}\sqrt{\frac{m}{\pi RT}}\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)^2} \tag{3}$$

Formulae (2) and (3) assume slightly different forms depending on the methods used in averaging the velocities of the gas particles. These have been discussed by Tait,[2] Rayleigh,[3] and Godfrey,[4] and it seems that the various methods yield expressions which differ from (2) and (3) by not more than 10 per cent. This difference is small enough to be ignored in the present case.

[1] Jeans, *Dynamical Theory of Gases*, p. 32.

[2] *Transactions Royal Society of Edinburgh*, **33**, 74, 1886.

[3] *Proceedings Royal Society*, *A*, **76**, 440, 1905. [4] *Loc. cit.*

*Comparison of theory and experiment.*—By giving values to $\nu$ corresponding to the various pressures $I$ was determined by means of (3) as a function of $\lambda$ for each pressure, and from this the width of the spectrum line was calculated for each pressure. At a temperature 0° C. and 760 mm pressure $\nu$ was taken to be $4\times10^{19}$. The diameter of a hydrogen molecule was $2\times10^{-8}$ cm. We assume that the gas was always at the temperature 600° C. $c=3\times10^{10}$, $R=9.3\times10^{-17}$, and $m=2.25\times10^{-24}$ grams. Introducing these values into (3) leads to

$$I=I_0\frac{\sin^2\frac{10634}{p}\left(\frac{1}{\lambda_0}-\frac{1}{\lambda}\right)}{\left(\frac{1}{\lambda_0}-\frac{1}{\lambda}\right)^2}, \qquad (4)$$

where $p$ is the pressure of the hydrogen in mm of mercury. When $I$ was computed from (5) for pressures within the range of the present measurements, curves of the type shown in Figure 3 were obtained. The width of the line increased with increase of pressure but was very narrow in all cases. For example, Figure 3 gives a curve for $\lambda_0=5000$ A and $p=100$ mm of mercury, and the width of the line represented by the curve is less than 0.04 A. In short, the broadening with pressure to be expected from the collision theory, or cause (3), is smaller by a different order of magnitude than that observed in the Balmer lines of hydrogen. In order to make the calculation agree approximately with the experimental measurements as far as the actual magnitude of the broadening is concerned, the number of equation (5) should be about 75 instead of 10634. The calculation was made on the basis

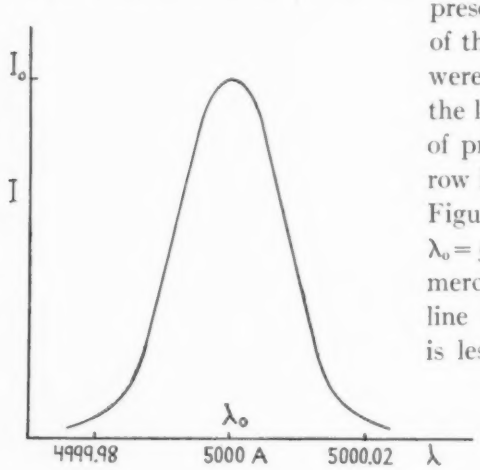


FIG. 3

of molecular hydrogen. Many researches, however, point to the fact that a disruptive discharge breaks the hydrogen molecule into its component atoms. If, then, we use values referring to the atoms, that is, employ a value for $\nu$ twice as great and for $\sigma$ and $m$ one-half as great as the molecular values previously substituted in (3), we find the number of equation (4) to be larger by the square root of two. This would make the calculated width of the line still smaller.

*Discussion.*—The discrepancy between theory and experiment is probably not to be found in experimental errors, approximate though the measurements were, or in a wrong assumption of the temperature of the gas. The discrepancy should rather be attributed to more fundamental errors of assumption, and in the light of the present experiment and the experiments of Merton and Nicholson (*loc. cit.*) we are ready to abandon cause (3) as inadequate and to turn to cause (6) as a possible explanation, as suggested by Merton (*loc. cit.*).

If the principal cause of the widening be attributed to the influence of an electric field on the radiating particle we may well inquire whether the electric field is the one imposed on the tube or is due to neighboring particles. Careful experiment on the separate influences of pressure and intensity of the discharge on the broadening combined with the known data of the Stark effect might differentiate between the two actions if they both exist. If the broadening is due to the electric field of neighboring particles we again return to the idea of collision. The effect of the collision upon the radiating particle, however, would be not merely a disturbance of the radiating centers by a mechanical shock but a profound momentary influence of an intense electric field.

STATE UNIVERSITY OF IOWA
IOWA CITY, IOWA
April 1922

# AN ESTIMATE OF THE DISTANCE OF THE ANDROMEDA NEBULA

By E. OEPIK

ABSTRACT

*Andromeda Nebula.*—Assuming the centripetal acceleration at a distance $r$ from the center is equal to the gravitational acceleration due to the mass inside the sphere of radius $r$, an expression is derived for the *absolute distance* in terms of the linear speed $v_0$ at an angular distance $\rho$ from the center, the apparent luminosity $i$, and $E$, the energy radiated per unit mass. From observations, $v_0$ comes out 157 km/sec. for $\rho = 150''$; and giving $i$ a value corresponding to magnitude 6.1, and assuming $E$ the same as for our Galaxy, the distance is computed to be 450,000 parsecs. This result is in agreement with that obtained by several independent methods. If it is correct, the *mass* within 150″ of the center is about $4.5 \times 10^9$ times the sun's mass, and the nebula is a stellar universe comparable with our Galaxy. *The ratio of the axes of the central ellipsoid*, whose shape is supposed to be due to rotation, was determined from photographs to be about 0.79.

Various estimates of the distance of the Andromeda Nebula have been made hitherto by H. Shapley,[1] H. D. Curtis,[2] K. Lundmark,[3] Luplau-Janssen and Haarh[4] and others; these estimates, based on the hypothesis that the Nebula consists of stellar matter similar to the matter of our Galaxy, lead to a distance of about $10^5$ to $10^6$ parsecs. Here we shall propose a method based on observed rotational movement, a method which may be applied to any nebula or cosmic system provided sufficient data are available. The principle is the same as that used by Campbell and Moore[5] to estimate the minimum mass of a planetary nebula.

Let $v_0$ be the velocity of motion along a circular orbit at a given point of the nebula, $r$, the distance from the center in astronomical units, $M$, the mass of the nebula within the radius $r(\odot = 1)$, $w$, the orbital velocity of the earth; then

$$\left(\frac{v_0}{w}\right)^2 = \frac{M}{r}. \qquad (1)$$

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 216, 1917.

[2] *Ibid.*, **29**, 206, 1917.

[3] *Astronomische Nachrichten*, **209**, 378, 1919.

[4] *Ibid.*, **215**, 285, 1922.

[5] *Lick Observatory Bulletin*, **9**, 4, 1916.

Let the integrated absolute luminosity ($\odot = 1$) of the nebula within the radius $r$ be $I$ and the apparent luminosity $i(\odot = 1)$; then

$$\frac{i}{I} = \frac{1}{D^2}, \tag{2}$$

$D$ being the distance in astronomical units. Let $\rho$ be the angular distance from the center of the nebula, so that $r = D \sin \rho$, and let $E = \frac{I}{M}$; then, combining (1) and (2), we obtain

$$D = \frac{E \sin \rho}{i}\left(\frac{v_0}{w}\right)^2; \tag{3}$$

$\rho$, $i$, and $w$ are known from observation; for $v_0$ an inferior limit is given by the relation $v_0 > \frac{v'}{\sqrt{2}}$, $v'$ being the velocity of rotation or, more generally, of any internal movement, observed spectroscopically. Thus the estimation of the distance depends chiefly upon the quantity $E$, representing the average loss of energy per unit mass; this quantity plays an important part in Eddington's theory of the radiative equilibrium of stars.

If there exists absorption of light in space, and if $\omega$ is the fraction of light transmitted, and $D'$ is the true distance, then the equations will be modified as follows:

$$\frac{i}{I} = \frac{\omega}{D'^2}; \tag{2'}$$

$$r = D' \sin \rho,$$

whence

$$D' = \frac{\omega E \sin \rho}{i}\left(\frac{v_0}{w}\right)^2 \tag{3'}$$

or

$$D' = \omega D, \tag{4}$$

$D$ representing the distance computed without absorption.

Hitherto no assumptions as to the minute structure of the nebula have been made; if we postulate the existence of stars of a given absolute luminosity, and if $a$ is their apparent luminosity computed without absorption, $a'$, the same quantity corresponding to the absorption factor $\omega$ and distance $D'$, then it is easy to obtain

$$a' = \frac{a}{\omega}. \tag{5}$$

Thus increased absorption diminishes the hypothetical distance but increases the apparent brightness of individual stars; this circumstance furnishes an upper limit of the hypothetical absorption, above which the individual stars of the nebula should become visible. Observational evidence indicates that if the Andromeda Nebula is a stellar universe with a luminosity-curve similar to that of our own stellar system, then $\omega$ cannot differ sensibly from unity.

For the Andromeda Nebula the following data may be used. F. G. Pease[1] has found a rotational velocity $v' = 72$ km/sec. at a distance of $\rho = 150''$ for which $\sin \rho = 0.000728$. To estimate the probable value of the circular velocity $v_0$, we shall suppose that the ellipsoidal shape of the inner parts of the nebula is a result of rotation. If $a$ and $b$ are the axes of the ellipsoid of revolution, $a > b$, $k = 1 - \frac{b}{a}$, then for small values of $k$ and for a rotating non-homogeneous gaseous mass condensed at the center we have approximately

$$k = \left(\frac{v}{v_0}\right)^2, \tag{6}$$

$v$ being the equatorial velocity of rotation. To obtain $k$, a plate taken with the 6-inch Petzval camera of the Dorpat Observatory, exposure 90 minutes, has been measured on a Hartmann microphotometer. Curves of equal density were constructed and the ratio of axes of these curves is given in the following table:

[1] *Proceedings of the National Academy of Sciences*, **4**, 21–24, 1918.

| Density of Negative | Millimeters | | $b:a$ |
|---|---|---|---|
| | $2a$ | $2b$ | |
| 5.00 | 0.87 | 0.69 | 0.79 |
| 4.60 | 1.12 | 0.88 | 0.79 |
| 4.30 | 1.40 | 1.02 | 0.73 |

Scale of the photograph: 1 mm = 261″.

These data give for $a = 150''$ the ratio $\frac{b}{a} = 0.79$ and $k = 0.21$. The angle between the line of sight and the equatorial plane of the nebula being small (about 8°) and the central condensation strong, we may assume that the values found represent the true ratio of axes of the ellipsoidal surfaces of equal density of the stellar or nebular matter within the nebula. Applying formula (6) with $k = 0.21$ and $v = v' = 72$, we obtain $v_0 = 157$ km/sec. This value must be regarded as an inferior limit, for in a stellar universe rotations in opposite directions may occur, and the spectroscopic velocity, representing the algebraic mean of all velocities, will be less than the mean absolute velocity.

As for $E$, we shall assume it equal to the corresponding value for our Galaxy. According to data given by Kapteyn and Van Rhijn,[1] 1000 cubic parsecs contain 44.7 luminous stars brighter than absolute magnitude 9.8, whose total luminous emission equals 55.5 stars of absolute magnitude zero[2] (nearly $= \odot$); J. H. Jeans[3] found that the observed motions of stars of our universe together with the distribution of density given by Kapteyn indicate that every luminous star counted by Kapteyn must correspond to an average mass equal to $3.2(\odot = 1)$; thus we obtain $E = \frac{55.5}{3.2 \times 44.7} = 0.38(\odot = 1)$ for our Galaxy.

The stellar magnitude of the Andromeda Nebula we shall assume = 5.0; the stellar magnitude of the sun = −26.6 (corresponding to absolute magnitude zero). From a paper by Reynolds,[4]

[1] *Mt. Wilson Contr.*, No. 188; *Astrophysical Journal*, **52**, 23, 1920.

[2] F. H. Seares, *Mt. Wilson Contr.*, No. 191; *Astrophysical Journal*, **52**, 167, 1920.

[3] *Monthly Notices*, **82**, 133, 1922.

[4] *Ibid.*, **74**, 132, 1914.

by a slight extrapolation, it has been found that the light of the nebula within 150″ from the center equals three-eighths of its total light; that corresponds to stellar magnitude 6.1. From these data we obtain $i = 2.512^{-32.7}$.

Substituting the data found above into equation (3), we find $D = 9.10^{10}$ astron. units or 450,000 parsecs. From equation (1) we obtain $M = 1.8 \times 10^9 (\odot = 1)$ within 150″ distance from the center. For a stellar system the mass within a given volume may be assumed proportional to the total luminous emission; thus the total mass of the Andromeda Nebula must be eight-thirds times greater or $4.5 \times 10^9$ times our sun's mass. This would correspond to about 1500 million of stars brighter than 1/10,000 of our sun, if the luminosity-curve and distribution of mass in the nebula were the same as in the regions of space surrounding us.

The order of the distance found above is in agreement with other estimates of the distance of the Andromeda Nebula; the coincidence of results obtained by several independent methods increases the probability that this nebula is a stellar universe, comparable with our Galaxy. It may be added that in our computation this statement did not enter as an initial hypothesis. The only assumption concerning the constancy of the energy-production $E$ may be true for any great aggregation of mass, without regard to the structure of the system; if we suppose that the energy of celestial bodies has its source in some processes like radioactivity, then our assumption would mean that the percentage of radioactive matter in the system of the Andromeda Nebula and in the Galactic system are equal.

TARTU (DORPAT)
April 7, 1922

# NOTE ON THE THICKNESS OF AIR REQUIRED TO PRODUCE THE ATMOSPHERIC ABSORPTION BANDS, A, *a*, AND B[1]

By ARTHUR S. KING

ABSTRACT

*Thickness of air required to give the absorption bands A, a, and B.—On a plate stained with dicyanin*, the A and B bands were obtained when the light from a tungsten lamp traversed minimum thicknesses of only 7 and 40 meters respectively; and though the air was hot and dry, the water band *a* at λ 7200 was obtained with absorption of 9.5 meters. To give the bands A and B visually respectively twelve and five times greater thicknesses of air are required.

In the course of experiments on absorption spectra in which the light from a 900-watt tungsten lamp passed through a heated graphite tube and thence to a 15-foot concave grating spectrograph, it was noted that the A band at λ 7600 frequently appeared on the spectrograms. This band is ascribed to the absorption of oxygen in the atmosphere. A test with the furnace cold showed that the absorption was due to the air at room temperature. The distance traversed by the beam through the air in the laboratory and in the spectrograph was about 12 meters. As this air-path is much shorter than that usually regarded as necessary to produce this band, other distances were tried in order to find, if possible, the minimum thickness required to give A, and also under what conditions the companion band B near λ 6900 could be obtained.

Placing the lamp as near the slit as could be conveniently arranged gave an air-path of 9.5 meters. The A band was still fairly strong and evidently would appear with a path considerably shorter. A surprising feature was that the water-vapor band a at λ 7200 could be detected at this distance. It was just at the limit of visibility on spectrograms having a favorable density of the continuous ground. As a part of the air-path was in the spectrograph pit, a determination of the humidity was not attempted; but the experiments were made on a particularly dry day, a hot wind blowing from the Mojave desert, so that the

[1] *Contributions from the Mount Wilson Observatory*, No. 232.

water-content of the air, both in the laboratory and in the ventilated pit, must have been very low.

Changing the air-path to 19 meters strengthened both A and *a*, but B was still invisible. The light was then passed the length of the laboratory to a plane mirror and back to the spectrograph, a distance of 39.4 meters. B now appeared, extremely faint, but the structure of both branches of the band could nevertheless be made out. Both A and *a* were strongly developed.

To find how small a layer of air would give A, it was necessary to use a shorter-focus spectrograph, and a 1-meter concave grating was tried. This involved a greatly reduced scale and, as the component lines of the atmospheric bands produced in this way are very fine, the matter of photographic resolution became important. Finally, by using a fine-grained film stained with dicyanin, the A band was detected with an air-path of seven meters.

These minimum distances of seven meters for A and approximately forty meters for B are much shorter than those given by Liveing and Dewar,[1] who extended the earlier observations of Egoroff and Janssen. Their tests, however, were visual, and the red-sensitive plate is certainly far more effective for the observations of A; while in the B region the plate probably still has an advantage over the eye in the detection of faint objects, on account of the possibility of adjusting contrast and the strength of the continuous ground.

Liveing and Dewar found that 7 atmospheres pressure in a steel tube 18 meters long rendered A visible, while 18 atmospheres were required to show B. The same tube filled with pure oxygen at atmospheric pressure showed A, while two atmospheres of oxygen were needed for B to appear. From this observation, A should be discernible through 90 meters of air. Egoroff[2] observed it through 80 meters of air, or nearly twelve times the amount found necessary in the writer's experiments by the photographic method.

MT. WILSON OBSERVATORY
November 1921

[1] *Proceedings of the Royal Society*, **46**, 222, 1889.

[2] *Comptes Rendus*, **101**, 1143, 1885.

## SPECTROPHOTOMETRIC METHODS FOR DETERMINING STELLAR LUMINOSITY. A CORRECTION

*Corrected parallax for Messier 11.*—In the *Publications of the Astronomical Society of the Pacific*, **34,** 115, 1922, Dr. K. Lundmark has corrected an unfortunate error in my paper, *Mt. Wilson Contr.*, No. 228; *Astrophysical Journal*, **55,** 85, 1922. For the cluster Messier 11 I found the modulus $M-m=-12$, which corresponds to the parallax 0".00040 and the distance 2500 parsecs or 8000 light-years (erroneously given 0".00025, 4000 parsecs, and 13,000 light-years).

BERTIL LINDBLAD

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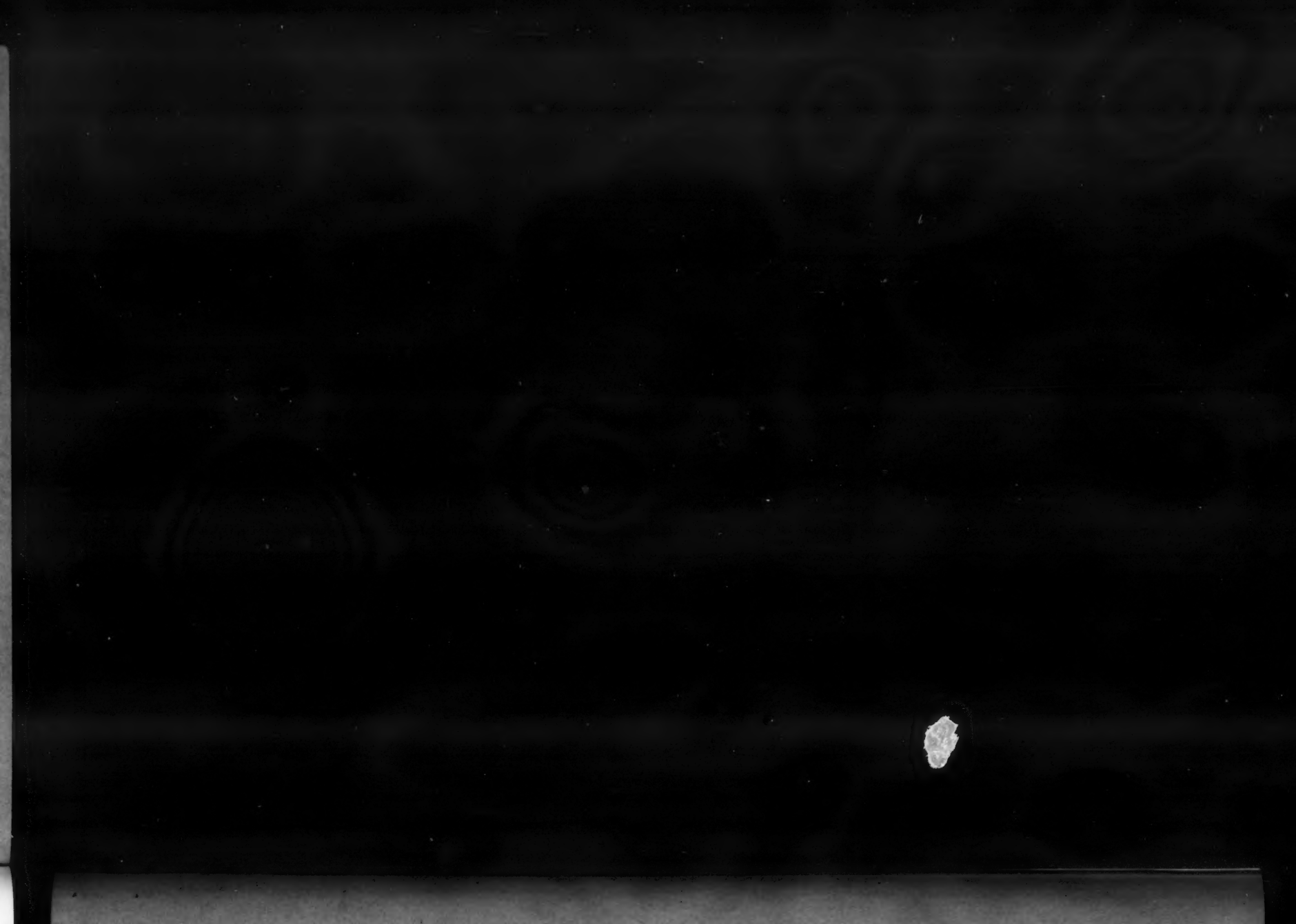
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